



Design of Reversible Logic based Basic Combinational Circuits

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ABSTRACT

With advances in VLSI technology, we are able to integrate more and more devices on single unit area to build more and more low power portable applications. As device size decreasing power dissipation becoming major concern for the design. In order to build low power systems there is a need for new technology which adapts logic which conserves energy and dissipates no power. Such upcoming technologies are quantum computing, quantum cellular automata, DNA computing, Optical computing and nano technologies. we have reversible circuits which dissipates no power or with zero internal power dissipation.. So there is a need for building combinational and sequential reversible circuits so that future computer can be replaced.. Attempt is made in this paper to build basic combinational logic circuits using reversible gates to minimize quantum cost, garbage outputs, ancilla inputs and gates. Results are verified using Xilinx14.2 tool with Spartan3 FPGA kit.

Keywords

Reversible, Garbage output, Quantum cost, Ancilla input

1. INTRODUCTION

As the technology is advancing we are building more and more portable devices by integrating more number of devices. Therefore energy dissipation is becoming key issue to be solved. R. Landauer in the year 1960 has demonstrated that energy dissipation due to information loss is high when circuits are constructed using irreversible logic. According to Landauers principle, the loss of one bit of information lost, will dissipate $kT \ln(2)$ joules of energy where, k is the Boltzmann constant and $k=1.38 \times 10^{-23}$ J/K, T is the absolute temperature in Kelvin [1]. The basic combinational circuits dissipate heat energy for every bit of information lost during the operation. This is because according to second law of thermodynamics, information once lost cannot be recovered by any methods. In 1973, Bennett, showed that in order to avoid $kT \ln(2)$ joules of energy dissipation in a circuit it must be built from reversible circuits [2]. According to Moores law the numbers of transistors will double every 18 months. Thus energy conservative devices are in need today. Reversible circuits are those circuits that do not lose information. The most important application of reversible logic lies in quantum computers [3]. A quantum computer will be viewed as a quantum network composed of quantum logic gates, with applications in various research areas like Low Power CMOS design, quantum computing, nanotechnology and DNA computing. Reversible computation in a system can be performed only when the system comprises of reversible gates. A gate is said to be reversible if the input vector can be uniquely recovered from the output vector and there is a one-to-one mapping between

its input and output assignments [4-5].The main challenges of designing reversible circuits are to reduce the number of gates, garbage outputs, constant inputs and quantum cost.

1.1 Basic Definitions related to reversible logic

Reversible logic gate

Reversible Gates are circuits in which number of outputs is equal to the number of inputs and there is a one to one correspondence between the vector of inputs and outputs [7]. It not only helps us to determine the outputs from the inputs but also helps us to uniquely recover the inputs from the outputs

Constant inputs:

This refers to the number of inputs that are to be maintaining constant at either 0 or 1 in order to synthesize the given logical function [8].

Garbage outputs:

Garbage is the number of outputs added to make an n-input k-output function reversible. We use the words constant inputs to denote the present value inputs that were added to an (n:k) function to make it reversible. The following simple formula shows the relation between the number of garbage outputs and constant inputs . Input + constant input = output + garbage. [6]

Quantum cost:

Quantum cost refers to the cost of the circuit in terms of the cost of a primitive gate. It is calculated knowing the number of primitive reversible logic gates (1×1 or 2×2) required to realize the circuit. The quantum cost of a circuit is the minimum number of 2×2 unitary gates to represent the circuit keeping the output unchanged. The quantum cost of a 1×1 gate is 1 and that of any 2×2 gate is the same, which is 1 [10].

Table I. List of Reversible Gates

| Reversible Gates | Size | Quantum cost | Functionality |
|------------------|------|--------------|---|
| NOT | 1X1 | 1 | $P=A'$ |
| CNOT(Feynman) | 2X2 | 1 | $P=A$ $Q=A \oplus B$ |
| Fredkin | 3X3 | 5 | $P=A$ $Q=A' \oplus B \oplus AC$ $R=AB \oplus A'C$ |

| | | | |
|----------------|-----|----|------------------------------------|
| Toffoli | 3X3 | 5 | P=A Q=B R=ABxorC |
| Pears | 3X3 | 4 | P=A Q=AxorB R=AB xor C |
| Double Feynman | 3X3 | 2 | |
| NG | 3X3 | 11 | P=A Q=AB xor C R=A'C' xor B' |

2. IMPLIMENTATION DETAILS

In order to design reversible combinational circuits, the conventional logic gates are appropriately designed from the reversible gates to reduce quantum cost, garbage outputs and constant inputs. Table 1 gives the list of reversible gates available and their functionality as well as quantum cost. Report also discusses possible alternative solution to realize the same function. Figures 1 & 2 shows the NAND realization using either fredkin gate or peres respectively. Figures 3 & 4 shows the AND function using Fredkin gate and peres gate respectively. Figures 5 & 6 shows OR and NOR function using fredkin gate. Figures 7,8 and 15 shows how single reversible gate is used to realize multiple conventional gates. Feynman Gates [8] are also used for copying the outputs and to avoid the fan out problem in reversible logic. In the Feynman gate, there are exactly two outputs corresponding to the inputs and a 0 in the second input will copy the first input to both the outputs of that gate. Hence, it can be concluded that Feynman gate is the most suitable gate for single copy of bit, as it does not produces any garbage output as shown in figures 13 and 14. Using these basic gates reversible cost effective combinational circuits are built and discussed in next section.

2.1 Proposed Basic And Universal Gates Using Reversible Gates

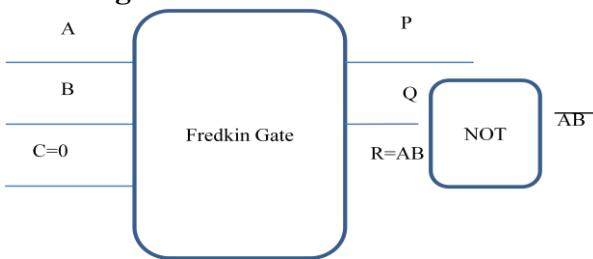


Fig. 1. NAND using Fredkin gate

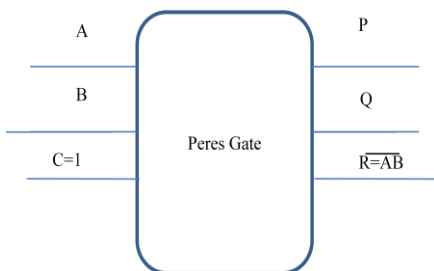


Fig. 2. NAND using Peres gate

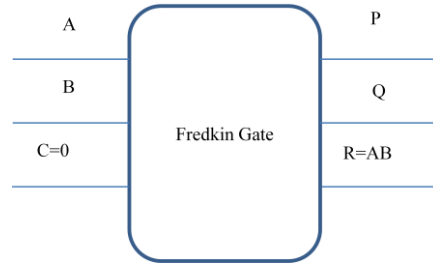


Fig. 3. AND using Fredkin gate

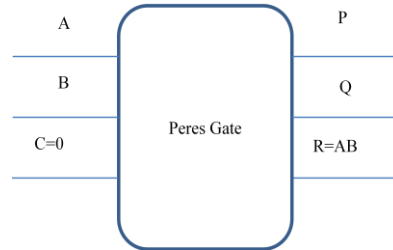


Fig.4. AND using Peres gate

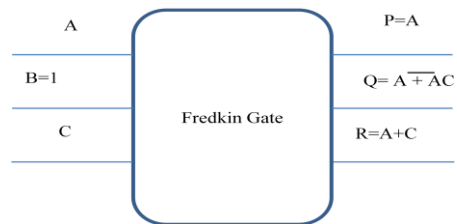


Fig.5. OR using Fredkin gate

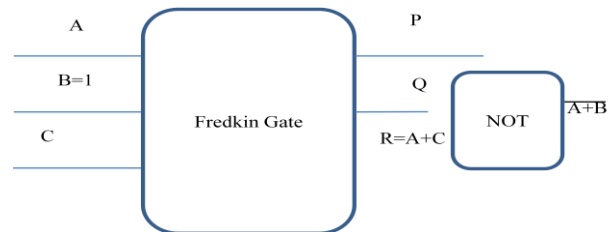


Fig.6. NOR using Fredkin gate

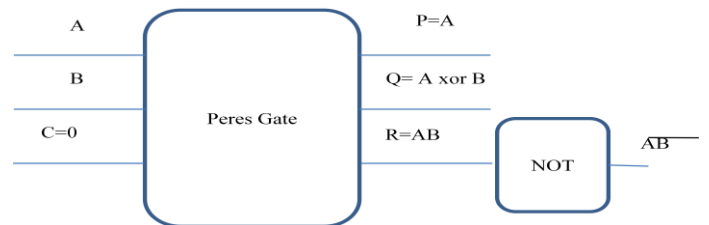


Fig.7. XOR, NAND and AND using Peres gate

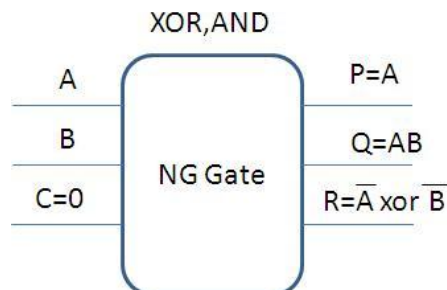


Fig.8. XOR and AND using NG gate

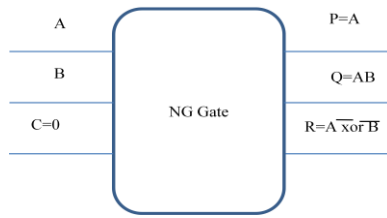


Fig.9. XOR and OR using NG gate



Fig.10. NAND and complement using NG gate

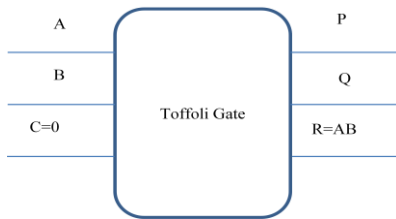


Fig.11. AND using Toffoli gate

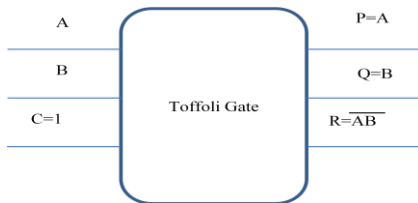


Fig.12. NAND using Toffoli gate

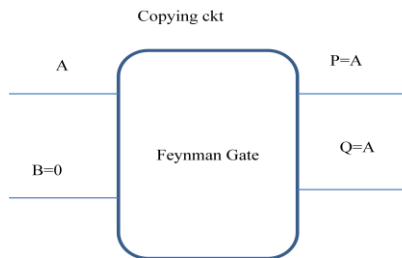


Fig.13. Feynman as a copying gate

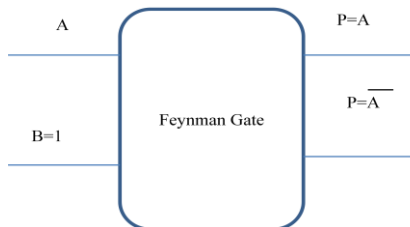


Fig.14. Feynman for complement gate

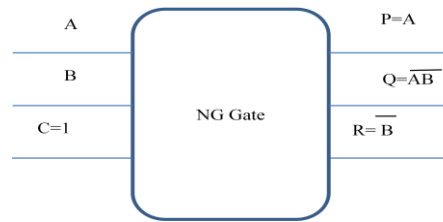


Fig.15. NG for NAND and complement gate

2.2 Proposed Combinational Circuits Using Reversible Gates

Adder and subtractor circuits are basic building blocks of any computing machine. Attempt is made here to realize cost effective reversible half, full adder and subtractors as shown in figures 16,17,18 and 19 respectively. Further 2:1, 4:1 multiplexers and 1:4 Demux circuits also realized using reversible gates as shown in figures 20,21,22 and 23. Design of 4:2 encoder and 2 bit comparator is shown in figure 24 and 25.

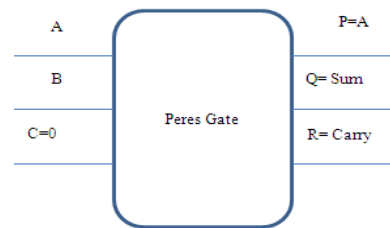


Fig.16. Half adder

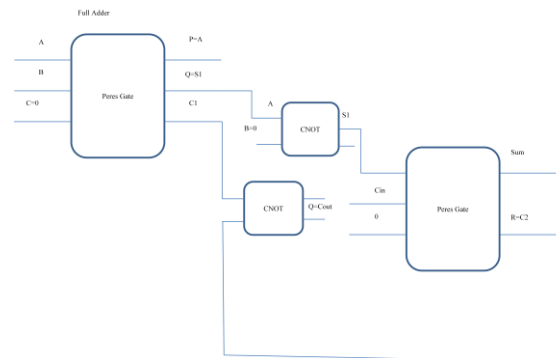


Fig.17. Full adder

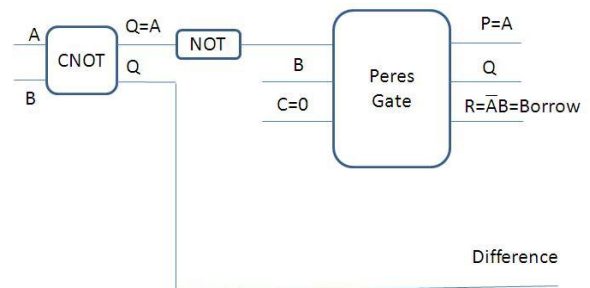


Fig.18. Half subtractor

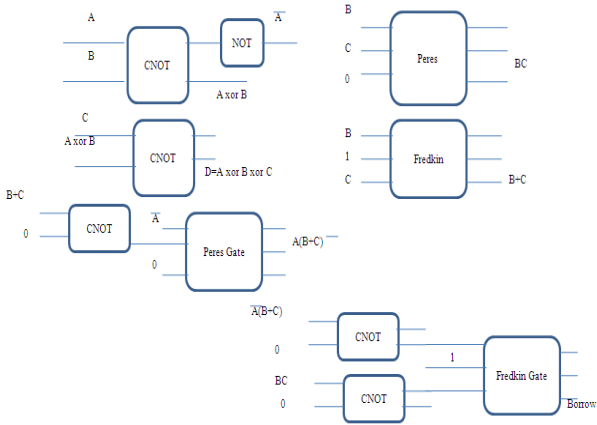


Fig.19. Full Subtractor

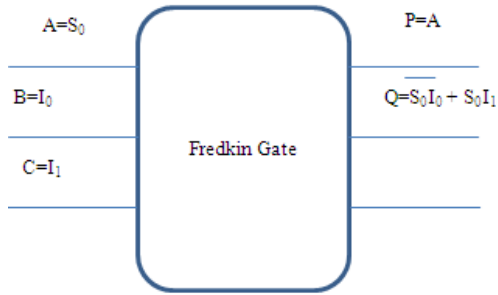


Fig.20. 2:1 mux

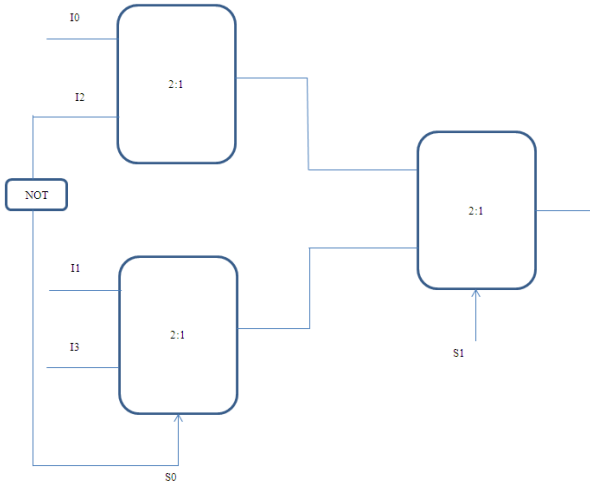


Fig.21. Conventional 4:1 mux

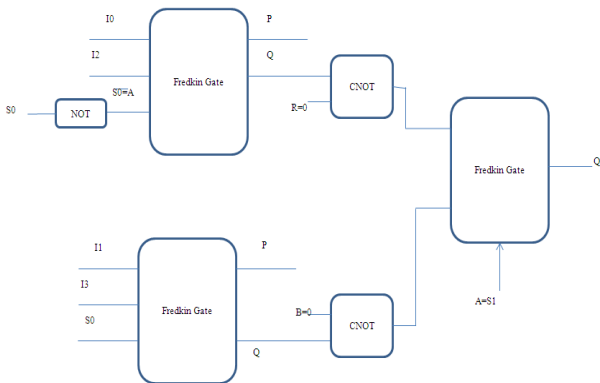


Fig.22. 4:1 Mux using reversible logic

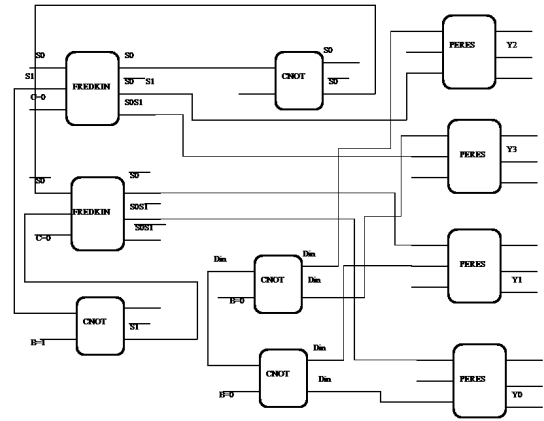


Fig.23. 1:4 Demux using reversible logic

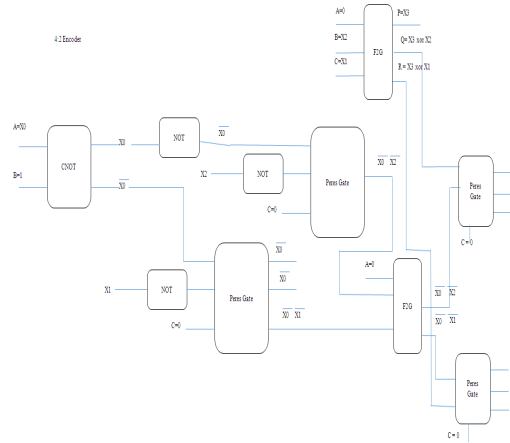


Fig.24. 4:2 Encoder

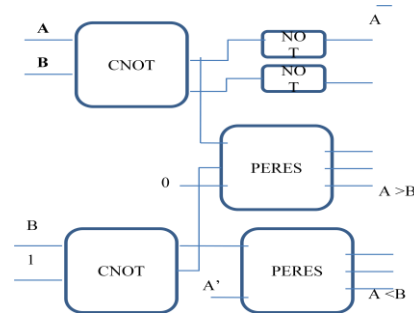


Fig.25. 2bit Comparator

3. RESULTS AND DISCUSSIONS

Proposed circuits using reversible gates are verified using Xilinx14.2 tool by developing verilog code and implemented on Spartan3 FPGA kit. Figures from 26 to 29 shows simulated results of half adder, full adder, half subtractor and full subtractor respectively. Simulated results of 2:1 and 4:1 mux is shown in figures 30 and 31. Figures 32 , 33 and 34 shows the results of 1:4 demux, 4:2 encoder and 2 bit comparator. TableII shows the summery all proposed circuits in this paper and respective performance parameters like quantum cost, garbage output and constant inputs. Attempt is made here to build combinational circuits using reversible gates which has minimum quantum cost.

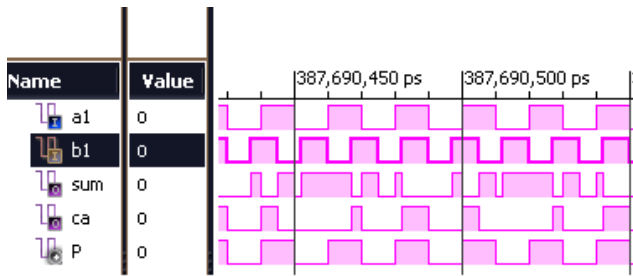


Fig. 26. Half Adder.

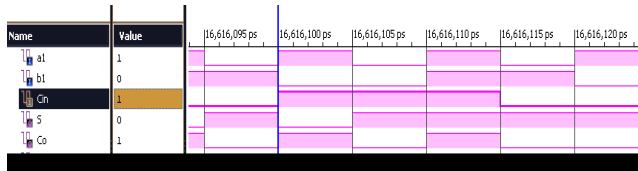


Fig. 27. Full Adder.

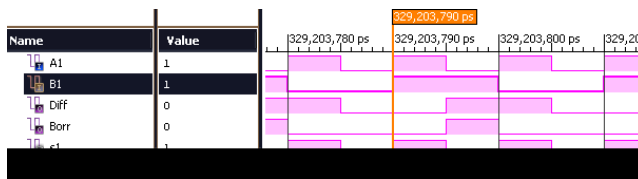


Fig. 28. Half Subtractor.

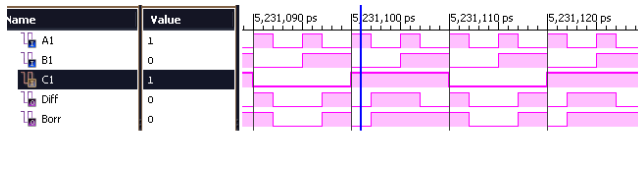


Fig. 29. Full Subtractor.

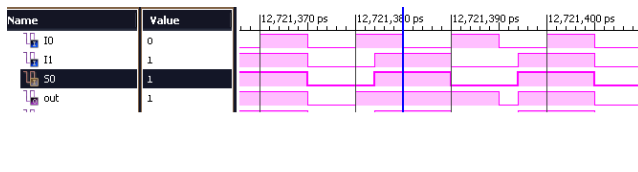


Fig. 30. 2:1 mux

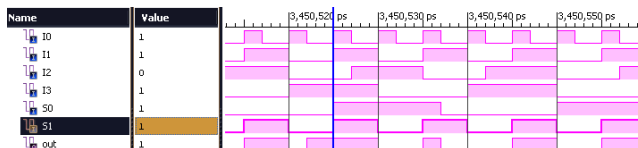


Fig. 31. 4:1 mux

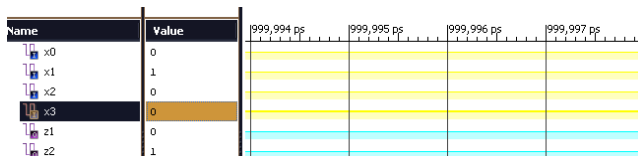


Fig. 32. 4:2 Encoder

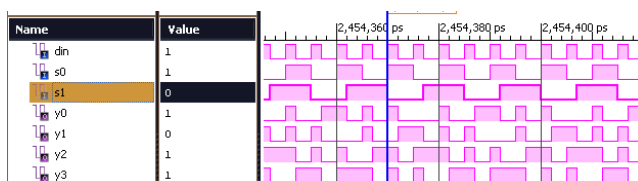


Fig. 33. 1:4 Demux

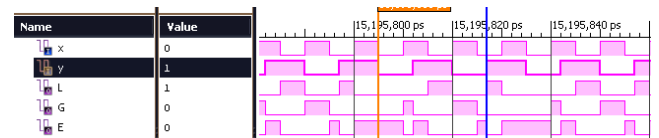


Fig. 34. 2 bit comparator

Table II. Summary of proposed circuits

| Logic circuits proposed | Reversible gates used | Quantum Cost | Garbage outputs | Input(auxiliary) constants |
|-------------------------|--------------------------------|--------------|-----------------|----------------------------|
| NAND | Peres | 4 | 2 | 1 |
| | Fredkin | 6 | 2 | 1 |
| AND | Peres | 4 | 2 | 1 |
| | Fredkin | 6 | 2 | 1 |
| OR | Fredkin | 5 | 2 | 1 |
| | Peres | 6 | 2 | 1 |
| XOR_AND_NAND | Peres,NOT | 5 | 1 | 1 |
| XOR_AND | NG | 11 | 1 | 1 |
| NAND,Complement | NG | 11 | 1 | 1 |
| XOR,NOR | NG | 11 | 1 | 1 |
| AND | Toffoli | 5 | 2 | 1 |
| NAND | Toffoli | 5 | 2 | 1 |
| Copying circuit | Feynman or CNOT | 1 | 0 | 1 |
| Half adder | Peres | 4 | 1 | 1 |
| Full adder | Peres,Cnot | 10 | 4 | 3 |
| Half Subtractor | Peres,Cnot | 6 | 2 | 1 |
| Full Subtractor | Cnot,not,Peres, Fredkin | 23 | 8 | 7 |
| 2:1 Mux | Fredkin | 5 | 2 | 0 |
| 4:1 Mux | Fredkin,Cnot | 17 | 7 | 2 |
| 4:2 Encoder | Cnot,double Feynman,Peres, Not | 24 | 9 | 6 |
| 1:4 Demux | Peres,Fredkin,C not | 30 | 11 | 10 |
| 2 bit comparator | Cnot,Peres,Not | 12 | 4 | 3 |

4. CONCLUSION

This paper proposes design of conventional basic, universal gates and few combinational circuits like adders, subtractors, mux/demux, encoder and comparators using reversible logic. Attempt is made to realize these with minimum quantum cost, garbage outputs and delay so that any complex digital applications can be made more optimized. All circuits are simulated using xilinx 14.2 and implemented on Spartan3 FPGA development kit. Paper also discusses how Feynman gate is used for generating complement output and as a copying circuit.

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