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Hanoi+: An Automated System to Solve Towers of Hanoi Puzzle

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ABSTRACT
This paper provides an extended research work on an embedded system named as Hanoi+ which is implemented to operate via smart phone over Bluetooth communications. The functionalities of Hanoi+ are based on Natural Algorithm. In practical state this robot is capable to provide solution for multiple towers and multiple disks problem by using minimum number of operations and minimum cost path. In this paper, provided solution is for six towers and five disks have been shown and the make the hardware configuration flexible enough to get a better performance.

General Terms
Artificial Intelligence, Robotics.

Keywords
Arduino Mega 2560, Natural Algorithm, Hanoi, Robotic Arm, Bluetooth, Android System.

1. INTRODUCTION
Tower of Hanoi is a popular mathematical puzzle which consists of three towers and multiple disks of different sizes. At the initial stage all disks are kept arbitrarily in a tower called as "source tower". The objective of the game is to shift the entire disk's stack to destination tower named as "target tower" according to the given size of the disks. During the movement the other tower can be used named as "rest tower". Also minimum number of steps will be required to transport disks [1-4]. The TOH puzzle has two major parts:

1. Number of movements of the disks for a particular combination of tower and disk.
2. Movements of the disks between the source towers to the target tower.

In case of 3 towers in the game, then the solution is quite straight forward, to solve the puzzle required minimum number of moves is 2^d - 1, where 'd' presents the quantity of disks. But with the increase of quantity of towers and disks, complexity of the problem and the algorithm size increases dramatically that introduces significant number of equations.

In previous paper, natural algorithm has been developed that provides the solution for TOH problem with finite number of towers and disks [5]. The main portion of Hanoi+ is a mechanical arm which is designed to capture a disk and to move it from one tower to another tower is an extended part of Hanoi [6] in terms of hardware specification. The arm is capable to move about vertical direction with the support of a stepper motor [7]. At the bottom surface of the arm, there is an electromagnet [8] attached to capture the disks which are made of steel.

To minimize the complexity of disk transportation a specific location is created for each of the tower on the periphery of a circle which is named as 'base'. In this project total 6 towers are used. That's why there is 60° angular displacement in between of two towers. The base is mounted on another stepper motor and capable to move in both of the clock-wise and counter-clock-wise directions. The arm is placed just at the periphery of it, so it lifts up & down a disk and the base rotates to change the tower.

2. RELATED LITERATURE
The Towers of Hanoi was originally invented by the French mathematician Eduard Lucas [9] in 1883. He was inspired by a legend that tells of a Hindu temple where the pyramid puzzle might have been used for the mental discipline of young priests.

There are two main aspects involved in the TOH puzzle and they are number of towers as well as the number of disks. There are so many software applications available in the form of video games [10]. Recently different robots are implemented to perform this puzzle physically. One of them is used a touch screen and 3 touch pins to move the disks from source tower to the destination tower [11]. Another robot is implemented, which have 3 towers on a linear path and having an arm with electromagnetic capabilities to transport disks [12] from source to destination. The mentioned robots are able to solve only 3 towers puzzle only.

In 2008 we have designed the robot Hanoi which was capable to solve 8 towers and 15 disks puzzle. This version [13] has an electromagnetic arm at the center of the circular path way and capable to move in both clock-wise and counter-clock-wise direction with the disk lifting properties. Later Hanoi was redesigned [14] for maximum 6 towers as well as 5 disks. In this version the mechanical arm has removed from the center of the towers path way and placed beside the circle where the arm is fixed in its position and has only the disk lifting capabilities. And also the base of towers has the capability to rotate in both clock-wise and counter-clock-wise directions.

3. NATURAL ALGORITHM (NA)
In the TOH, there are two variables- 'tower' and 'disk'. For a specific number of towers the number of disks may vary as the numbers of steps are different. The core of NA 'f(t, d)' provides the right solution according to the given quantity where the entire algorithm is classified into some parts for a specific number of towers and also a specific number of disks.

Let's consider, 't' and 'd' are two variables namely towers and disks respectively. These two are defined as $(0 < t = n)$ and $(0 < d = m)$ where 'n' and 'm' represents the maximum

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number of the towers and disks respectively. In Hanoi+ ‘n’ is considered 6 and ‘m’ is considered as 5.

The algorithm provides three different situations, depending on the combination of towers and disks provided to the Hanoi+, shown in table 2.

3.1 f(t, d) = 2^d – 1 when, t=3

This situation arises when the numbers of tower are exactly three at any positions out of six. Here only two towers are engaged at a time t_l & t_r or t_t & t_r or t_l & t_t, where t_t presents the temp tower, t_r presents the right tower and t_l presents the left tower.

3.2 f(t, d) = 2d – 1; when, t>d

When the numbers of tower are greater than the number of given disks, then this formula is applicable to calculate the number of maximum movements. It also indicates that all the disks must be moved twice except the largest one.

3.3 f(t, d) = 4d – 2t + 1; when, t≤d≤(t(t-1)/2)

This formula is required to calculate the number of movements for the rest of the combinations. In this case some disks require four movements and some are two. But the largest one needs only one step to go to the destination.

Table 1. Total number of movements in f(t, d)

d\t	t_0	t_1	t_2	t_3	t_4	t_5
d_0	-	1	1	1	1	1
d_1	-	-	3	3	3	3
d_2	-	-	7	5	5	5
d_3	-	-	15	9	7	7
d_4	-	-	31	13	11	9

4. NA BASED HANOI$^+$ SYSTEM

NA provides the number of transportations with necessary directions. The following algorithm is presenting the minimum cost to shift a disk from one tower to another:

Set the index of Source tower into S_t

Set the index of Destination tower into D_t

Keep the clockwise distance of S_t and D_t into C_c

Keep the Anti-clockwise distance of S_t and D_t into C_{ac}

if, C_c is less than C_{ac} then

Traverse clockwise

else

Traverse Anti-Clockwise

Steps that require computing a TOH puzzle consist of four towers and four disks is 9, shown in table 2.

Table 2. Steps required for 4 towers and 4 disk puzzle

Steps	t_0	t_1	t_2	t_3
0	$d_3 > d_2 > d_1 > d_0$			
1	$d_3 > d_2 > d_1$			d_0
2	$d_3 > d_2$		d_1	d_0
3	d_3	d_2	d_1	d_0
4	d_3	d_2	$d_1 > d_0$	
5		d_2	$d_1 > d_0$	d_3
6			$d_1 > d_0$	$d_3 > d_2$
7	d_0		d_1	$d_3 > d_2$
8	d_0			$d_3 > d_2 > d_1$
9				$d_3 > d_2 > d_1 > d_0$

4.1 System Structure of Hanoi$^+$

Proposed system provides the output in the real world by moving the stack of disks maintaining all the terms and conditions. At the initial stage the arm is set at rest angle, assuming that it’s 0° of the circular path way. The towers are placed at the appropriate positions. Then the disks are placed at the source tower.

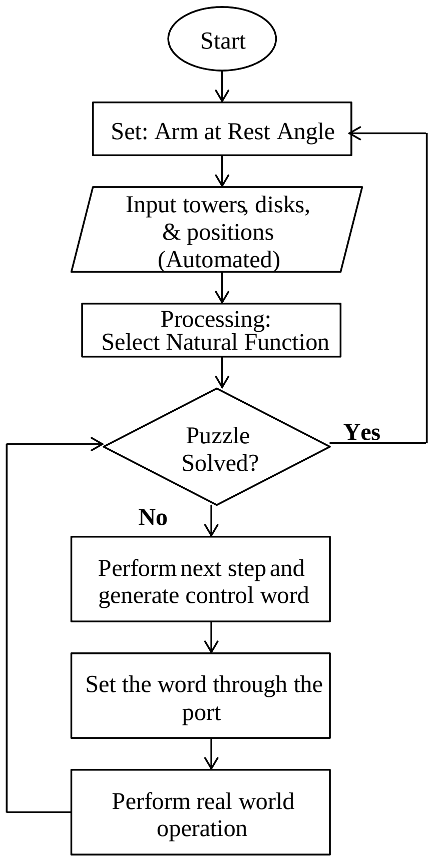

Fig 1: System model of Hanoi$^+$

After placing the disks a command is given that ‘m’ numbers of disks are placed at z^{th} tower position meaning the source tower. The present system is capable to get command in two different ways, such as the physical keyboard as well as app keyboard meaning virtual keyboard. After getting the human command, it classified the problem according to the type of the puzzle, thru the core of NA ‘f (t, d)’. Then the whole command is broken down into some steps as equal to the number of total movements obtained from f (t, d).

The central function initiates a two dimensional array to hold the number of disks and towers of the puzzle and also classify the puzzle to solve using the appropriate function. Later it sends one by one disk movement 'Control Word' shown in Fig 1. Normally the arm is placed at the top most position to maintain the maximum height. When a disk is needed to be lifted from source tower it is moved towards the downward direction. When the top of the disk is sensed by the floor contact sensor, the motion of the arm is stopped and the electromagnet is energized to capture the disk.

After capturing the disk, the arm is moved towards upward direction and stopped right after getting the maximum height. Then the base starts to rotate towards the destination tower. Once the destination tower is reached just below the arm, it started to move towards downward direction to place it at the destination tower.

4.2 Machine Interfacing and Operation

At the early stage, system is designed for Intel Pentium IV machine where communication took place through parallel port (LPT 1). At the improvisation of hardware design then we introduced ATmega8 [15] microcontroller with MUX [16] and later we replaced Atmega8 with Arduino Uno R3 [17] to obtain 32kb memory instead of 8kb, without this others configuration doesn't have significant differences. According to system communication demand it requires 25 I/Os where Arduino UNO R3 is only capable to provide 20 I/Os. That implies still it has lacking of 5 I/Os. So we introduced a MUX of 16 inputs to fulfill the requirements.

At present the core of the entire system is Arduino Mega 2560 having microcontroller ATMEGA2560 [18] which is used to control the whole system that provides 25 bits information for communication purpose. The microcontroller receives total 16 bits of information from 4 different types of sensors. Previously we had used a MUX that obtained 16 bits inputs from the sensors attached at different positions where towers provide 6 bits, disks provide 6 bits, ARM and its body provides 3 bits, and tower base provides 1bit input.

The latest system does the same task as a built-in service just by replacing the MUX. There are total 6 optical sensors is used to detect the towers. The sensors are arranged such a way that when a tower is placed in any of the 6 places the corresponding sensor will provide logic '1' to notify that a tower is placed at that position otherwise the sensor will provide logic '0'. The output of these sensors is used as the input of the digital channels of Arduino Mega 2560 from 22 ~ 27.

Again 6 optical sensors are used to detect the disk stack. The sensors are arranged such a way that when disk stack is placed in any of the 6 places the corresponding sensor will provide logic '1' to notify that the disk stack is placed at that position otherwise the sensor will provide logic '0'. The output of these sensors is used as the input of the Arduino Mega 2560 from 28 ~ 33. Disk stack positions will be detected from the input of signals as previous. Below Fig 2 represents the layout for circuit and other components of Hanoi.

Depending on the signals available from the digital pins of 22 ~ 33 of the Arduino Mega 2560 the system program generates the appropriate control word and then sends it to the machine. After getting this control word it is classified according to the machine requirements. 8 bits of the word is distributed for the stepper motors. 1 bit is allocated to handle the electromagnet attached at the bottom of the ARM. Block diagram of hardware operation is shown in Fig 3.

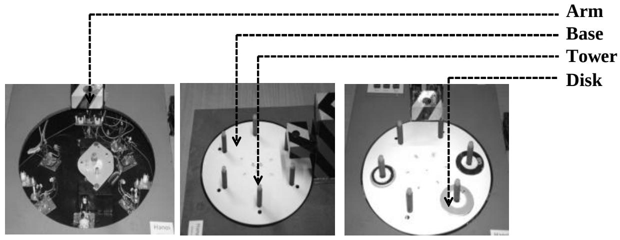

Fig 2: Layout of circuit and other components of Hanoi+

In the previous system there is a command keyboard, from where a user can insert the necessary instruction to the machine. The latest version is designed such a way that it is capable to get the command also from smart phones. To develop these advanced facilities we have used HC-05 Bluetooth Shield. The RX-TX of the Bluetooth module is connected to the digital pins 1 and 0 respectively. For the command purpose we have used an app titled 'Broxcode' [19] by which we have got the full access of the system in an android based smart phone.

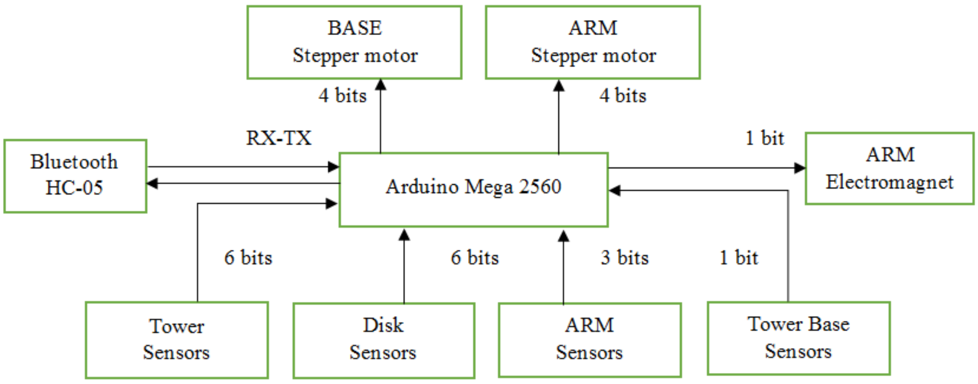

Fig 3: Block diagram of bitwise operation

5. LIMITATION AND FUTURE WORK

TOH is a very popular mathematical puzzle that requires special tools to solve it. In this paper we have represented an efficient process for multiple towers and disks problem by using minimum number of cost path.

For the reason of hardware limitations, our project is capable of only 6 towers and 5 disks; however the NA algorithm is capable of 16 towers and 15 disks. The positions of these 6 towers are fixed. Using any three of these positions, multiple combinations can be created for Hanoi$^+$. Thus it would be capable to demonstrate the critical part of TOH puzzle in many different ways which would be helpful to the learners. The existing systems are capable only to demonstrate the traditional version of TOH puzzle.

Also the current system has been developed to operate it from a smart phone, based on android operating system where three virtual buttons are available to provide the necessary command. This Bluetooth based communication has also providing an opportunity to operate the machine thru voice command in future over smart phone. This opportunity of interacting thru voice would be more interesting to operate Hanoi$^+$.

The TOH puzzle is bounded by some strict conditions like the must be kept in the form of stacks according to their size from larger to smaller. But a robot should be capable to sort a disks stack meaning the human operator would provide a number of disks to solve it. After getting the disks the robot will sort out the disks stack according to their sizes and then will start to move to the destination tower.

6. ACKNOWLEDGMENTS

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7. REFERENCES

[1] Janssen G, De Mey H, & Egger J (2009). Executive functioning in college students: Evaluation of the Dutch Executive Function Index (EFI-NL). International Journal of Neuroscience, 119, 792-805.

[2] Koorland MA & MacLeod S (2005). The accuracy improvement measure: A tool for assessing the effectiveness of teacher preparation programs. Journal of Precision Teaching and Celeration, 21, 13-18.

[3] Egger J, De Mey H, & Janssen G (2007). Assessment of executive functioning in psychiatric disorders: functional diagnosis as the ouverture of treatment. Clinical Neuropsychiatry, 4, 111-116.

[4] Cicerone K, Levin HS, Malec J, Stuss D, & Whyte J (2006). Cognitive rehabilitation interventions for executive function: Moving from bench to bedside in patients with traumatic brain injury. Journal of Cognitive Neuroscience, 18, 1212-1222.

[5] A. S. Zaforullah Momtaz, Md. Fayyaz Khan, Muhammad Sajjad Hossain, Aloke Kumar Saha & Kazi Shamsul Arefin. Design and Implementation of an Efficient Natural Algorithm to Solve Towers of Hanoi Puzzle having Multiple Towers & Disks. IACSIT International Journal of Engineering and Technology, Vol. 3, No. 4, August 2011.

[6] A. S. Zaforullah Momtaz, Md. Fayyaz Khan, Aloke Kumar Saha & Kazi Shamsul Arefin. A Practical Solution for Robotic Arm of the Towers of Hanoi Problem. International Journal of Computer and Electrical Engineering, Vol. 3, No. 4, August 2011.

[7] Stepper Motor: wikipedia.org/wiki/Stepper_motor

[8] Electromagnet: science.howstuffworks.com/electromagnet.htm

[9] TOH History: global.britannica.com/topic/Tower-of-Hanoi

[10] Online Game: www.superkids.com/aweb/tools/logic/towers

[11] TOH Robot: www.youtube.com/watch?v=ETNpYCq2AN8

[12] TOH Robot: www.youtube.com/watch?v=h8JPtWpctl4

[13] TOH Robot: www.youtube.com/watch?v=g2EepAQKhFo

[14] A S Zaforullah Momtaz, Nahida Sultana Chowdhury, S M Salim Reza. Implementation of an Embedded System Hanoi for Tower of Hanoi Puzzle. International Journal of Advances in Computer Science & Its Applications – IJCSIA Volume 4: Issue 4, 27 December, 2014.

[15] Atmega8 Microcontroller: www.atmel.com/devices/atmega8.aspx

[16] MUX: wikipedia.org/wiki/Multiplexer

[17] Arduino UNO R3: www.arduino.cc/en/Main/ArduinoBoardUno

[18] Arduino Mega 2560: www.arduino.cc/en/Main/arduinoBoardMega2560

[19] App to Operate Hanoi$^+$: play.google.com/store/apps/details?id=com.broxcode.arduinobluetoothfree