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Super Vertex Graceful Graphs

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ABSTRACT
For a defined graph labeling, there exists a number of bijective functions for a graph of defined order and size which leads to different graphs. In this paper, a mathematical tool is developed to find the number of super vertex graceful graphs for a defined order "p" and size "q".

Mathematics Subject Classification:
05C78-

Keywords
Order of a graph, size of a graph, graceful graphs, super vertex graceful graphs.

1. INTRODUCTION
In this paper, graph means a simple undirected graph. Labeling of a graph is the process of assigning values to vertices or edges or both subject to certain conditions. This concept of labelling was introduces by A. Rosa in the name of β- valuation in 1960s. Later. It was named as graceful labelling by Golomb. Since then many types of labelling came into existence. A detailed report on the labelling of graphs is given by Gallian [1]. In 2009, Sin Min Lee and others [9] defined super vertex graceful labelling and analysed the super vertex gracefulness of unicyclic graphs under this labelling. N. Murugesan, R. Uma [4,5,6] have analysed complete bipartite graphs, twigs, spiders, regular caterpillars, fire cracker graphs under the same labeling. and some properties of super vertex graceful graphs. In this paper, a model is defined to find the number of super vertex graceful graphs for defined order "p" and size "q".

2. DEFINITION
2.1 Super vertex graceful labelling:
A graph G with p vertices and q edges , vertex set V(G) and edge set E(G), is said to be super vertex graceful (SVG), if there exists a function pair (f, f^+) where f is a bijection from V(G) onto P, f^+ is a bijection from E(G) onto Q, such that $f^+(u,v) = f(u) + f(v)$ for any $(u,v) \in V(G)$, where P and Q are finite set of integers defined and as follows:

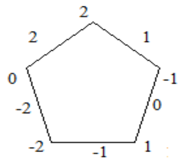

Figure 1 Super vertex graceful graph C_5

$$P = \begin{cases} \pm1, \pm2, \dots, \pm\frac{p}{2} \ if\ p\ is\ even \\ 0, \pm1, \pm2, \dots, \pm\frac{p-1}{2} \ if\ p\ is\ odd. \end{cases} \quad Q = \begin{cases} \pm1, \pm2, \dots, \pm\frac{p}{2} \ if\ q\ is\ even \\ 0, \pm1, \pm2, \dots, \pm\frac{p-1}{2} \ ifq\ is\ odd. \end{cases}$$

2.2. Example
In the graph given in fig 2, both the size and order is 5. Therefore P = Q = { -2,-1,0,1,2}. Here f^+ and f are defined such that $\mathbf{f^+}$ (-1,1)=0; $\mathbf{f^+}$ (-1,2)=1; $\mathbf{f^+}$ (0,2)=2; $\mathbf{f^+}$ (1,-2)=-1; $\mathbf{f^+}$ (-2,0)=-2.Then G is SVG. It is interesting to note that C_5 is SVG. But Golomb [2], discussed that C_5 is not graceful.

3. RESULT
In labeling, many types of graphs exist for a defined order and size. Similarly, for a particular graph, there exist different types of graphs with same type of labeling. In this section this concept is explained in the context of super vertex graceful labeling.

3.1 Theorem
Let G be a graph of order 'p' and size 'q'. Then by the definition of super vertex graceful graphs the vertex set label 'P' and edge set label 'Q' are

$$P = \begin{cases} \pm1, \ \pm2 \ , \dots \pm\frac{p}{2} ifp\ is\ even \\ 0, \pm1, \pm2, \dots \pm\frac{p-1}{2} ifpisodd \end{cases}$$

$$Q = \begin{cases} \pm1, \ \pm2 \ , \dots \pm\frac{q}{2} ifqiseven \\ 0, \pm1, \pm2, \dots \pm\frac{q-1}{2} ifqisodd. \end{cases}$$

The edge set labels are obtained from the vertex labels. The number of ways of obtaining the edge label +k and –k are the same. The following set of ordered pairs induces the particular edge label.

The set of ordered pairs depends on the size of the graph (odd or even)

In order to form super vertex graceful graph of order 'q' (odd) we have to select one order pair from each row of table 1. The number of ways selection of order pairs of label 'i' is α_i for each label i. Therefore the edge label 0 can be obtained from α_0 ways. If the order pair selected is (1,-1), for the edge label 0, the order pair for edge label '1' can be obtained

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from α_1 order pairs, similarly, the edge label 2 can obtained from α_2 pairs etc. So, the number of ways of obtaining the edges 1, 2, … $\frac{q-1}{2}$ is $\alpha_1 . \alpha_2 \ldots . \alpha_{\frac{q-1}{2}}$. Similar argument can be applied for the edge labels -1, -2, … $-\frac{q-1}{2}$. Hence the number of super vertex graceful graphs is $2. \alpha_0 . \alpha_1 . \alpha_2 \ldots . \alpha_{\frac{q-1}{2}}$ if 'q' is odd.

Also, if 'p' is even $0 \notin P$ and the corresponding number of ordered pairs reduces by '1' for all the edge labels in the second column of the tables 1 and 2.

3.2 Example

Let us consider a graph of order 6 and size 10. By the definition of super vertex graceful map, vertex label P and edge label set Q are defined as $P = \{\pm 1, \pm 2, \pm 3\}$ and $Q = \{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$.The ordered pairs that induce the edge labels from the given vertex labels are given in the table 3. The vertex pairs (-1, 3), (1,2), (1,3), (2,3), (1,-3), (-1, -2), (-1, -3), (-2, -3) belong to all the set of ordered pairs given in the second column.

3.3Remark

The super vertex graceful graphs obtained from the above given methodology is connected only if the union of the elements of the ordered pairs is equal to the vertex label set P. Otherwise the graph is disconnected.The above concept is illustrated by an example.

Let us consider the graph of order '7' and size '9'. Then $P = \{\pm 1, \pm 2, \pm 3\}$ and $Q = \{0, \pm 1, \pm 2, \pm 3, \pm 4\}$. The induced edge labels and the set of ordered pairs are tabulated in table 4.

Let us consider the following set of ordered pairs which induce the labels 0, 1, 2, 3, 4, -1, -2, -3, -4 be (1,-1), (0,1), (-1,3), (0,3), (1,3),(0,-1), (0,-2),(0,-3), (-1,-3). In the above set of ordered pairs the vertex label 2 is missing. The following graph with this set of vertex label is shown below and it is disconnected.

Table 1 Edge labels and the set of ordered pairs if 'q' is odd

Edge Label	Set of ordered pairs	No. of ordered pairs
0	(1,-1), (2, -2), (3,-3) ,…$\left(-\left(\frac{q-1}{2}\right), \left(\frac{q-1}{2}\right)\right)$	α_0
1	(0,1), (-1, 2),(-2,3) ,….$\left(-\left(\frac{q-3}{2}\right), \left(\frac{q-1}{2}\right)\right)$	α_1
2	(0,2), (-1, 3),(-2,4), ….$\left(-\left(\frac{q-5}{2}\right), \left(\frac{q-1}{2}\right)\right)$	α_2
…	…	
…	…	
…	…	
$\frac{q-1}{2}$	$\left(0, \left(\frac{q-1}{2}\right)\right)$	$\alpha_{\frac{q-1}{2}}$
-1	(0,-1), (1,-2),(2,-3) ,….$\left(\left(\frac{q-3}{2}\right), -\left(\frac{q-1}{2}\right)\right)$	α_1
-2	(0,-2), (1, -3),(2,-4), ….$\left(\left(\frac{q-5}{2}\right), -\left(\frac{q-1}{2}\right)\right)$	α_2
…	…	…
…	…	…
$-\left(\frac{q-1}{2}\right)$	$\left(0, -\left(\frac{q-1}{2}\right)\right)$	$\alpha_{\frac{q-1}{2}}$

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Table 2. Edge labels and the set of ordered pairs if 'q' is even

Edge Label	Set of ordered pairs	No. of ordered pairs
1	(0,1), (-1, 2),(-2,3), ….$\left(-\left(\frac{q-2}{2}\right),\left(\frac{q}{2}\right)\right)$	α_1
2	(0,2), (-1, 3),(-2,4), ….$\left(-\left(\frac{q-4}{2}\right),\left(\frac{q}{2}\right)\right)$	α_2
…	…	…
…	…	…
$\frac{q}{2}$	$\left(0,\left(\frac{q}{2}\right)\right)$	$\alpha_{\frac{q}{2}}$
-1	(0,-1), (1,-2),(2,-3), ….$\left(\left(\frac{q-2}{2}\right),-\left(\frac{q}{2}\right)\right)$	α_1
-2	(0,-2), (1, -3),(2,-4), ….$\left(\left(\frac{q-4}{2}\right),-\left(\frac{q}{2}\right)\right)$	α_2
…	…	…
…	…	…
$-\left(\frac{q-1}{2}\right)$	$\left(0,-\left(\frac{q}{2}\right)\right)$	$\alpha_{\frac{q}{2}}$

In order to form super vertex graceful graph of order 'q' (odd) we have to select one order pair from each row of table 1. The number of ways selection of order pairs of label 'i' is α_i for each label i. Therefore the edge label 0 can be obtained from α_0 ways. If the order pair selected is (1,-1), for the edge label 0, the order pair for edge label '1' can be obtained fromα_1 order pairs, similarly, the edge label 2 can obtained from α_2 pairs etc. So, the number of ways of obtaining the edges 1, 2, …$\frac{q-1}{2}$ is$\alpha_1.\alpha_2 \ldots.\alpha_{\frac{q-1}{2}}$. Similar argument can be applied for the edge labels -1, -2, …$-\frac{q-1}{2}$. Hence the number of super vertex graceful graphs is $2.\alpha_0.\alpha_1.\alpha_2 \ldots.\alpha_{\frac{q-1}{2}}$ if 'q' is odd.

Also, if 'p' is even $0 \notin P$ and the corresponding number of ordered pairs reduces by '1' for all the edge labels in the second column of the tables 1 and 2.

4.5.2 Example:

Let us consider a graph of order 6 and size 10. By the definition of super vertex graceful map, vertex label P and edge label set Q are defined as $P = \{\pm1, \pm2, \pm3\}$ and $Q = \{\pm1, \pm2, \pm3, \pm4, \pm5\}$.The ordered pairs that induce the edge labels from the given vertex labels are given in the table 3.

Table 3. Edge labels and the set of ordered pairs if q = 10

Edge Label	Set of ordered pairs	No. of ordered pairs
1	(-1, 2),(-2, 3)	2
2	(-1, 3)	1
3	(1, 2)	1
4	(1,3)	1
5	(2,3)	1
-1	(1,-2),(2,-3)	2

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-2	(1, -3)	1
-3	(-1,-2)	1
-4	(-1,-3)	1
-5	(-2, -3)	1

Table. 4 List of ordered pair inducing required edge labels for q= 10

Proper choice of labels	Corresponding ordered pairs that induce the label	No. of ways
(1, 2, 3, 4, 5,-1, -2, -3-4,-5)	(-1,2), (-1,3), (1,2), (1,3), (2,3), (1,-2), (1,-3), (-1,-2), (-1,-3), (-2, -3)	2
	(-2,3), (-1,3), (1,2), (1,3), (2,3) (1,-2), (1,-3),(-1,-2), (-1,-3), (-2, -3)	
(1, 2, 3, 4,5,-1, -2, -3-4,-5)	(-1,2), (-1,3), (1,2), (1,3), (2,3) (2,-3), (1,-3),(-1,-2), (-1,-3), (-2, -3)	2
	(-2,3), (-1,3), (1,2), (1,3), (2,3) (2,-3), (1,-3),(-1,-2), (-1,-3), (-2, -3)	
Total		2 x 2 = 4

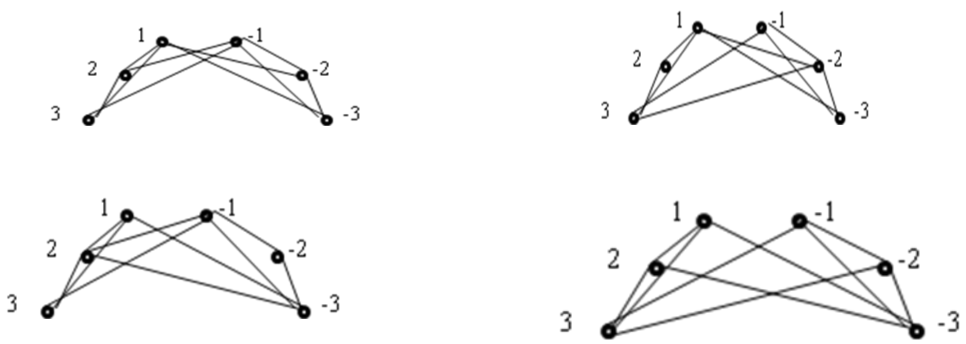

Figure 2 Super vertex graceful graphs of order 6 and size 10

The vertex pairs (-1, 3), (1,2), (1,3), (2,3), (1,-3), (-1, -2), (-1, -3), (-2, -3) belong to all the set of ordered pairs given in the second column.

4.5.2.1Remark

The super vertex graceful graphs obtained from the above given methodology is connected only if the union of the elements of the ordered pairs is equal to the vertex label set P. Otherwise the graph is disconnected.The above concept is illustrated by an example.

Let us consider the graph of order '7' and size '9'. Then $P = \{\pm1, \pm2, \pm3\}$and $Q = \{0, \pm1, \pm2, \pm3, \pm4\}$. The induced edge labels and the set of ordered pairs are tabulated in table 5.

Table 5. Edge labels and the set of ordered pairs if q = 9

Edge Label	Set of ordered pairs	Number of ordered pairs
0	(1,-1), (2,-2), (3,-3)	3
1	(0,1), (-1, 2), (-2, 3)	3
2	(0, 2), (-1, 3)	1

3	(0,3), (1, 2)	1
4	(1,3)	1
-1	(0,-1),(1,-2),(2,-3)	3
-2	(0,-2), (1, -3)	1
-3	(0, -3),(-1,-2)	1
-4	(-1,-3)	1

Let us consider the following set of ordered pairs which induce the labels 0, 1, 2, 3, 4, -1, -2, -3, -4 be (1,-1), (0,1), (-1,3), (0,3), (1,3),(0,-1), (0,-2),(0,-3), (-1,-3). In the above set of ordered pairs the vertex label 2 is missing. The following graph with this set of vertex label is shown below and it is disconnected.

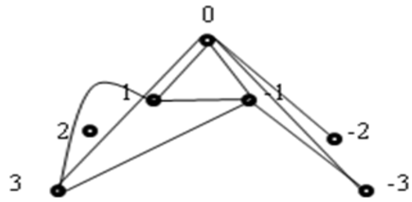

Figure 3. Disconnected super vertex graceful graph

4. ACKNOWLEDGMENTS

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