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Efficient Underdetermined DOA Estimation Algorithm by Extending Covariance Matrix based on Non-Circularity using Coprime Array

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ABSTRACT
Real world signals are non-stationary but can be modeled as stationary within the local time frames. These types of signals are called quasi stationary signals (QSS). In this paper a Khatri-Rao (KR) subspace based direction of arrival (DOA) estimation of QSS is considered by designing a coprime array structure. This structure provides an alternative way to enhance the degrees of freedom (DOF) and it can also eliminate mutual coupling effects. One of the most important observations is that the covariance matrix can be extended based on non-circularity of QSS. The covariance matrix exhibits non-circularity due to the non-circular behavior of QSS. Exploiting the non-circularity an extended covariance matrix (ECM) is designed to achieve higher DOF. Hence, the proposed algorithm has the capability to uniquely estimate DOA's more than twice the number of sensors. Simulation results show that the proposed algorithm can achieve better performance as compared to Khatri-Rao (KR) subspace, coprime array with displaced arrays (CADiS) and nested array based techniques under various situations.

Keywords
Quasi stationary signals, Khatri-Rao, extended covariance matrix, degrees of freedom, mutual coupling, and direction of arrival.

1. INTRODUCTION
Direction of Arrival (DOA) estimation is a vital part in array signal processing. Precise estimation of DOA is an indispensable part of many real world applications like radar, microphone array systems, sonar and speech processing. Over the decades, a number of sophisticated techniques have been developed like well-known multiple signal classification (MUSIC) [1], [2],estimation of signal parameters via rotational invariance technique (ESPRIT) [3] and their variant algorithms [4-6]. These techniques are eminently known as sub-space (SS) techniques.

The main concern in this paper is to estimate DOA's of quasi-stationary signals (QSS) considering Khatri-Rao (KR) technique. Most of real world signals like speech and audio are non-stationary. However, exploiting their second order statistics (SOS) they can be modeled as stationary signals by modeling them locally static over a small interval of time (this interval is called a frame) [7]. So, real world signals like speech and audio signals can be modeled as QSS signals which provides a strong inspiration to study DOA estimation of QSS signals. The KR sub-space approach has some distinctive benefits, like an underdetermined scenario can be converted into different virtual overdetemined cases. Secondly, it provides an efficient way to enhance aperture size which eventually help us to estimate more number of users than sensors and lastly, it can effectively eliminate spatial noise from the received SOS of signals without knowing the noise covariance and even it is also effective for colored noise. In [7], a KR MUSIC based DOA estimation technique was developed considering QSS signals using uniform linear array (ULA) in order to estimate more number of sources than sensors (underdetermined case). A low complexity ESPRIT technique was designed using KR [8]. And this concept was further extended considering L-shaped and uniform circular array in [9] and 10]. However, the conventional array unable to achieve higher degrees of freedom (DOF). In order to achieve high DOF, [11] designed a nested array and they showed that considering N antennas they can achieve $O(N^2)$DOFs after applying KR technique. However, closely spaced antennas intrinsically generate mutual coupling which degrades the performance of the system. In order to avoid mutual coupling while attaining higher DOFs Co-Prime array [12] offers a better alternative.
In this paper, a coprime array structure is proposed. In section 2 two coprime arrays are concatenated in such a way that no array element overlap physically. In order to achieve large DOF as obtained by nested array, a KR representation of cross covariance matrix (CCM) is developed in section 3 and then this CCM further extended based on non-circular property.

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Finally, the DOA estimation is obtained by applying MUSIC technique. Section 4 comprises with extensive simulations. Compared to KR approach, CADiS based method [13] and nested array based technique, the proposed method provides better DOA estimation performance and remarkably able to estimate DOA's more than twice the number of sensors.

2. DATA MODEL

Consider a coprime array design using two co-prime numbers *M* and *N* and elements are aligned along positive and negative x-axis in such a way that positive axis contains *M* elements with inter element spacing $N\lambda/2$ and negative axis contains *N*-1 number of elements after removal of common element with inter element spacing $M\lambda/2$. The configuration of the proposed structure of coprime array which consist of *M*+*N*-1 elements shown in Fig.1. Assume *K* uncorrelated narrow band, far field speech signals (QSS) impinging on the array, the output of the array can be expressed as

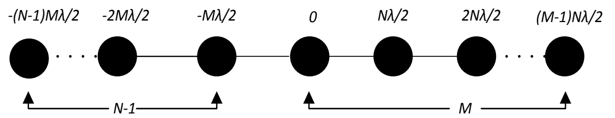

Fig.1 Design of Coprime array structure

$$\boldsymbol{y}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

where $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \dots, \boldsymbol{a}(\theta_K)]$ is the steering matrix and θ_k is the DOA of k^{th} source. And $\boldsymbol{a}(\theta_k)$ is the steering vector which can be expressed in compact form as $\boldsymbol{a}(\theta_k) = [e^{j(N-1)M\pi \sin\theta_k}, \dots, e^{jM\pi \sin\theta_k}, 1, e^{jN\pi \sin\theta_k}, \dots, e^{j(M-1)N\pi \sin\theta_k}] \in \mathbb{C}^{(M+N-1)\times K}$, $\boldsymbol{s}(t) = [s_1(t), s_2(t), \dots, s_k(t)]^T$ is zero mean QSS signals vector and $\boldsymbol{n}(t) \in \mathbb{C}^{M+N-1}$ is the spatial noise. For F number of frames $\boldsymbol{s}(t)$ can be modeled as QSS signals (under a certain assumption that their second order statistics remain static over a small period of time) which satisfy wide-sense stationary (WSS) condition within a frame length Las

$$E[|s_k(t)|^2] = d_{fk}, \forall\, t \in [(f-1)L, fL-1] \tag{2}$$

where,f is the frame index and Eq. (2) depicts SOC of QSS is time varying but remains static over a short period of time.

3.1 Local Covariance matrix design

Under the stationarity assumption local covariance matrix can be designed as

$$\boldsymbol{R_f} = E\{\boldsymbol{y}(t)\boldsymbol{y}^H(t)\} \in \mathbb{C}^{M'\times M'}, \forall\, t \in [(f-1)L, fL-1] \tag{3}$$

where, $f = 1,2,\dots,F$ denotes the frame index and $M' = M + N - 1$. Local covariance matrices (LCM) can be estimated by averaging locally as follows

$$\boldsymbol{R_f} = \left(\frac{1}{L}\right)\sum_{t=(m-1)L}^{mL-1} \boldsymbol{y}(t)\boldsymbol{y}^H(t) \tag{4}$$

The above equation can be represented in matrix form as

$$\boldsymbol{R_f} = \boldsymbol{A}\boldsymbol{D}_f\boldsymbol{A}^H + \boldsymbol{C} \tag{5}$$

where, $\boldsymbol{D_f} = Diag(d_{f1}, d_{f2}, \dots, d_{fK})\,\} \in \mathbb{C}^{K\times K}$ is the source covariance matrix per frame, $\boldsymbol{A}$ is the steering matrix and $\boldsymbol{C}$ is the spatial noise matrix. Hence, having$F$ number of local covariance matrices $\boldsymbol{R}_1, \boldsymbol{R}_2, \dots, \boldsymbol{R}_F$the objective is to estimate DOAs $\theta_1, \theta_2, \dots, \theta_k$ from $\boldsymbol{R}_1, \boldsymbol{R}_2, \dots, \boldsymbol{R}_F$ without knowing the information of spatial noise $\boldsymbol{C}$and local source covariance matrix$\boldsymbol{D_f}$.

3. DOA ESTIMATION USING KR SUBSPACE TECHNIQUE

The vectorization of LCM using KR technique can be represented as

$$\boldsymbol{y}_f = vec(\boldsymbol{R_f}) = vec(\boldsymbol{A}\boldsymbol{D}_f\boldsymbol{A}^H) + vec(\boldsymbol{C}) \tag{6}$$

$$\boldsymbol{y}_f = (\boldsymbol{A}^*\odot\boldsymbol{A})\boldsymbol{d}_f + vec(\boldsymbol{C}) \quad f = 1,2,\dots,F \tag{7}$$

Hence, by stacking $[\boldsymbol{y}_1, \boldsymbol{y}_2, \dots, \boldsymbol{y}_F]= \boldsymbol{Y}$

$$\boldsymbol{Y} = (\boldsymbol{A}^*\odot\boldsymbol{A})\phi^T + vec(\boldsymbol{C})\boldsymbol{1}_F^T \tag{8}$$

where, $\odot$ represent KR product, $\phi = [\boldsymbol{d}_1, \boldsymbol{d}_2, \dots, \boldsymbol{d}_F]^T$ each column of ϕrepresents source power vector and$\boldsymbol{n} = vec\ (\boldsymbol{C})$ represent noise vector respectively. The most important observation in the above equation is that the physical steering vector of size $M' \ll K$ has been converted into $\boldsymbol{A} = (\boldsymbol{A}^*\odot\boldsymbol{A}) \in \mathbb{C}^{(M'M')\times K}$after designing a virtual steering matrix using KR technique. Now this underdetermined case has been converted into overdetermined case using KR technique which provides an opportunity to estimate more number of sources than sensors. The second advantage is that considering coprime array, there is no overlapping in virtual elements due to discrete spacing which ultimately remove the angle ambiguity problem.

Considering the quasi stationary condition, noise covariance can be eliminated by using orthogonal complement projector $\boldsymbol{P}_{1F}^{\perp} = \boldsymbol{I}_F - (1/F)\boldsymbol{I}_F\boldsymbol{I}_F^T$ as

$$\boldsymbol{Y}\boldsymbol{P}_{1F}^{\perp} = [(\boldsymbol{A}^*\odot\boldsymbol{A})\phi^T + vec(\boldsymbol{C})\boldsymbol{1}_F^T]\boldsymbol{P}_{1F}^{\perp}$$

$$= (\boldsymbol{A}^*\odot\boldsymbol{A})(\phi^T\boldsymbol{P}_{1F}^{\perp}) = \boldsymbol{A}_{eff}\boldsymbol{B} \tag{9}$$

where, $\boldsymbol{A}_{eff} = (\boldsymbol{A}^*\odot\boldsymbol{A})$and $\boldsymbol{B}$=$(\phi^T\boldsymbol{P}_{1F}^{\perp})$ is of full rankunder the assumption of matrix full column rank as mentioned in [7] respectively. Orthogonal projection does not damage rank of the covariance matrix and the advantage of the noise eliminator is that it does not change the dimension of final covariance matrix.

3.2 Extended Covariance Matrix design by exploiting non-circularity

Inspired by non-circular (NC) signals [14] and [15], CCM also exhibit similar NC property as follows

$$E[(\boldsymbol{s})(\boldsymbol{s})^H] \neq \boldsymbol{0} \tag{10}$$

$$E[(\boldsymbol{s})(\boldsymbol{s})^T] = \rho e^{-j\beta}E[(\boldsymbol{s})(\boldsymbol{s})^H] \tag{11}$$

where,$0 \le \rho \le 1$,$\beta$is circularity rate and phase respectively. We consider $\rho = 1$means only non-circular signals. This non-circular nature of the signal can be utilized to enhance DOF of the proposed algorithm just like [16] and [17] where they enhanced DOF by considering non-circular signals. One of the most important observation is that $\boldsymbol{Y}\boldsymbol{P}_{1F}^{\perp}$ and its conjugated counter-part are non-overlapped (except at 0^{th} position) and distinct. This observation paved away to design an extended covariance matrix to further enhance degrees of freedom. Hence, the extended covariance matrix can be written as

$$\boldsymbol{R}_{ext} = \begin{bmatrix} (\boldsymbol{Y}\boldsymbol{P}_{1F}^{\perp}) \\ (\boldsymbol{Y}\boldsymbol{P}_{1F}^{\perp})^{\dagger} \end{bmatrix} = \begin{bmatrix} (\boldsymbol{A}_{eff}\boldsymbol{B}) \\ (\boldsymbol{A}_{eff}\boldsymbol{B})^{\dagger} \end{bmatrix} \quad (12)$$

Alternatively,

$$\boldsymbol{R}_{ext} = \begin{bmatrix} \boldsymbol{A}_{eff} \\ (\boldsymbol{A}_{eff})^{\dagger} \end{bmatrix} \boldsymbol{B} = \boldsymbol{A}_{ext}\boldsymbol{B} \quad (13)$$

where $\boldsymbol{R}_{ext} \in \mathbb{C}^{(2M'M'-1)\times K}$ is the extended covariance matrix and $(\boldsymbol{Y}\boldsymbol{P}_{1F}^{\perp})^{\dagger}$ is the conjugated counter-part after removal of 0^{th} reference point and $\boldsymbol{A}_{ext} = [\boldsymbol{A}_{eff}, (\boldsymbol{A}_{eff})^{\dagger}]^T$ is extended steering vector.Hence, KR based virtual steering vector along with modified covariance matrix collectively exhibits non circular property which ultimately help us to calculate more number of DOAs.

3.3 MUSIC based DOA estimation

In order to extract noise subspace, singular value decomposition (SVD) can apply on $\boldsymbol{R}_{ext}$. The SVD of $\boldsymbol{R}_{ext}$ can be represented as

$$\boldsymbol{R}_{ext} = [\boldsymbol{U}_s \boldsymbol{U}_n] \begin{bmatrix} \Sigma_s \boldsymbol{0} \\ \boldsymbol{0}\boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_s^H \\ \boldsymbol{V}_n^H \end{bmatrix} \quad (14)$$

where, $\boldsymbol{U}_s \in \mathbb{C}^{(M'M')\times K}$, $\boldsymbol{V}_s \in \mathbb{C}^{F\times K}$, $\boldsymbol{U}_n \in \mathbb{C}^{M'^2\times(M'^2-K)}$ and $\boldsymbol{V}_n \in \mathbb{C}^{F\times\left(M'^2-K\right)}$ are the left and right singular values associated with the non-zero singular values. Similarly, $\boldsymbol{U}_n \in \mathbb{C}^{M'^2\times(M'^2-K)}$ and $\boldsymbol{V}_n \in \mathbb{C}^{F\times(M'^2-K)}$ are the left and right singular values associated with the zero singular values respectively, and $\Sigma_s \in \mathbb{C}^{K\times K}$ is a diagonal matrix containing nonsingular values. Hence, using MUSIC our objective is to find θ_k where, $k = 1,2,\dots,K$ such that

$$\boldsymbol{U}_n{}^H[(\boldsymbol{A}^*\odot\boldsymbol{A})]_k = \boldsymbol{U}_n^H\left(\boldsymbol{a}^*(\theta_k) \otimes \boldsymbol{a}(\theta_k)\right) = \boldsymbol{0},$$

$$\theta = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad (15(a))$$

$$\boldsymbol{f}(\boldsymbol{\theta_k}) = \frac{\boldsymbol{1}}{\boldsymbol{U}_n^H(\boldsymbol{a}^*(\boldsymbol{\theta_k})\otimes \boldsymbol{a}(\boldsymbol{\theta_k}))(\boldsymbol{a}^*(\boldsymbol{\theta_k})\otimes \boldsymbol{a}(\boldsymbol{\theta_k}))^{\boldsymbol{H}}\boldsymbol{U}_n} \quad (15(b))$$

Table I. Summary of the proposed Algorithm

Given: coprime array structure, received signal sequence $\{\boldsymbol{y}(\boldsymbol{t})\}_{\boldsymbol{t=0}}^{\boldsymbol{T-1}}$, source number K and frame length L.

Step 1. Compute the local covariance estimates per frame using Eq. (4-6).

Step 2. Concatenate data vectors after applying vectorization

$$\mathbf{Y}=[\boldsymbol{y}_1, \boldsymbol{y}_2, \dots, \boldsymbol{y}_F]$$

Step 3. Apply noise covariance elimination using Eq. (9).

Step 4. Design extended covariance matrix $\boldsymbol{R}_{ext}$ based on non-circular property using Eq. (12).

Step 5. Perform SVD on $\boldsymbol{R}_{ext}$ and extract noise subspace $\boldsymbol{U}_n \in \mathbb{C}^{M'^2\times(M'^2-K)}$

Step 6. Perform MUSIC DOA spectrum search using Eq. (15) and pick the larges peaks of the above function as DOAs of the transmitted sources.

4. SIMULATION RESULTS

In this section several sets of simulations are provided to demonstrate the performance of the proposed method. The root mean square error (RMSE) is used to check the performance of the proposed method which is defined as

$$RMSE = \frac{1}{K}\sum_{k=1}^{K}\sqrt{\frac{1}{MC}\sum_{m=1}^{MC}\left[\hat{\theta}_{k,m} - \theta_k\right]^2} \quad (15)$$

where, K is the total number of sources, MC is number of Monte-Carlo simulations, $\hat{\theta}_{k,m}$ is the estimated DOA of a particular k^{th} source at m^{th} MC simulation and θ_k is the original DOAof k^{th} source respectively.

A six sensors coprime array considering two coprime numbers M=3 and N=4 is taken. The sources are distributed uniformly. The QSS signals are generated randomly (see TABLE II in [7]) considering the frame length randomly generated following a uniform distribution on [300,700]. However, fixed frame length of L=512 is chosen. The Simulation results are divided into two cases: 1st which deals with DOA spectrum and 2nd deals with RMSE performance of different and proposed methods against SNR, number of frames etc.

CASE I: In this experiment 13 sources ($>2K$) which was distributed uniformly over the range of [-75 75] are taken and these sources were collected using a Coprime array of six sensors and averaged over 1000 Monte-Carlo simulations. Fig.2 depicts the DOA spectrum against angles at SNR=15dB. The figure shows that our proposed algorithm can uniquely and precisely detect all DOAs even it is an underdetermined case.

One of the most important observation is that sources greater than twice the number of sensors can be detected. This observation proves that covariance matrix shows non-circular property (which finally help us to extend array aperture along with KR technique).

CASE II: In all the following experiments, eight uncorrelated uniformly distributed QSS sources (generally L=512 and F=50) are considered with a coprime array consisting of six sensors. For a fair comparison QSS signals are taken for all techniques. The grid step is taken 1° degree and 1000 MC simulations was performed for every experiment.

In this experiment the proposed algorithm is compared with nested array MUSIC [11], KR MUSIC [7] and CADiS based MUSIC [13]. Fig.3 depicts the performance comparison of the proposed algorithm along with other techniques against SNR considering same number of sensors. The RMSE performance of proposed algorithm is quite better as compared to the rest of the algorithms. Even our proposed algorithm outperform nested array based MUSIC due to the reason that large inter element spacing eliminated the mutual coupling effect which ultimately enhances the error performance.

Fig.4 shows RMSE performances of proposed algorithm against number of frames F considering different methods. The frame length is fixed at L=512 and the SNR is 14dB.The figure shows that our proposed method outperform other methods (KR MUSIC, Nested array based MUSIC and CADiS based MUSIC) which shows the superiority of proposed technique.The important observation is that RMSE performance increases rapidly as the number of frames increasesas compared to other methods.

Lastly, Fig.5 shows the RMSE performance of the proposed algorithm against number of frames at different SNR values. Clearly, RMSE performance became better and better after increasing number of frames or/and SNR values.

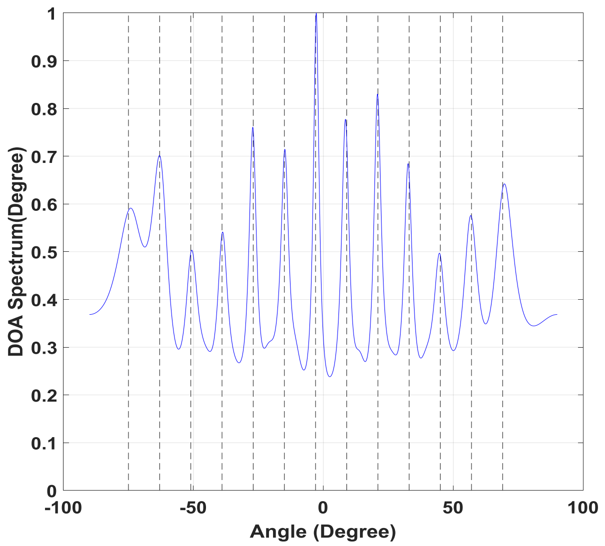

Fig.2 DOA spectrum

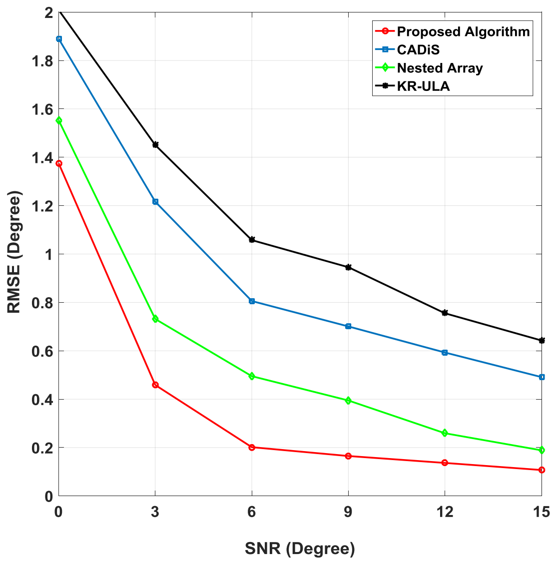

Fig.3 Performance comparison of different methods

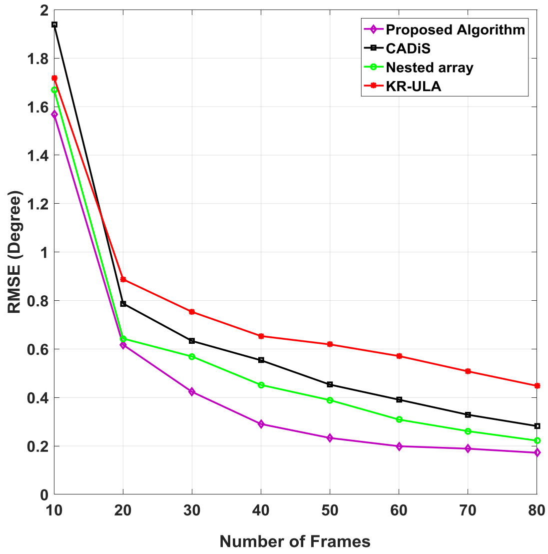

Fig.4 Performance comparison of different no. of frames

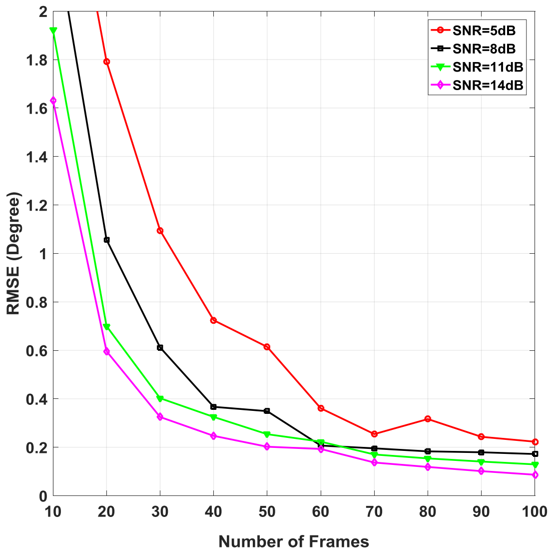

Fig.5 Performance comparison of different no. of frames

5. CONCLUSION AND PROSPECTIVE PLAN

In this paper, a novel DOA estimation algorithm is presented which has the capability to estimate more than twice the number of sources. This capability was achieved by extending the covariance matrix based on non-circular property after applying KR technique. Moreover, proposed algorithm is robust to mutual coupling affects and also solves angle ambiguity problem. Lastly, it showed better DOA performance in term of RMSE under different scenarios.

In future the prior focus will be how to increase DOFs by achieving large number of lags considering displaced co-prime arrays and how to fill the lags to achieve better consecutive lags to enhance MUSIC based estimation.

6. ACKNOWLEDGMENTS
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