Communications on Applied Electronics (CAE) – ISSN : 2394-4714
Foundation of Computer Science FCS, New York, USA
Volume 7 – No. 11, December 2017 – www.caeaccess.org

Inverse Line Domination Number of Jump Graph

N. Pratap Babu Rao
Department of Mathematics
S.G. Degree College KOOPAL 583231, Karnataka, India

ABSTRACT
Let J(G) =(V,E) be a jump graph. Let D is minimum line dominating set in a jump graph E(J(G)). If E-D contains a line dominating set D' of E(J(G)) then D' is called an inverse dominating set with respect to D. The cardinality of an inverse line dominating set of a jump graph J(G) is called inverse line dominating set of E(JG)).

In this paper we study theoretic properties of inverse line domination of jump graph and its exact value for some standard graphs. The relation between inverse line domination of jump graph with other parameters is also investigated.

Keywords
Graph circumference, diameter, domination, inverse line domination number, jump graph.

1. INTRODUCTION
Let G(p,q) be a graph with p=|V| and q=|E| denotes the vertices and edges of a graph G respectively.

All the graphs considered here are finite , non-trivial, undirected and connected without loops or multiple edges. For basic terminology we refer to Chartrand and Lesinik[7]. A set D of edges in a graph G is a line dominating set if every edge in WE-D is adjacent to some edge in D. The line domination number $\gamma^{'}(G)$ of G is the minimum cardinality of a line dominating set of G

Let d be a minimum line dominating set of G. if E-D contains a line dominating set D' of G then D' is called inverse line dominating set with respect to D'. The inverse dominating number γ^{-1}(G) of G is the minimum cardinality of an inverse dominating set of G. This concept was introduced by Kulli and Sigarkanti[3] and it was studied by several graph theorists in [8,9]

In general the degree of vertex v in a graph G is the number of edges of G, incident with v and it is denoted by degv. The maximum (minimum) degree among the vertices of G is denoted by Δ(G) (δ(G)). We denote the minimum number of edges in edge cover of G (i.e., edge cover number) by $\alpha_1(G)$ and the minimum number of edges in independent set of edges of G(i.e., edge independent set by $\beta_1(G)$. The subgraph induced by $X \subseteq V$ is denoted by <X>. A vertex of degree one is called an pendent vertex. A vertex adjacent to pendent vertex is called the support vertex. The maximum d(u,v) for all u in G eccentricity of v and the maximum d(u, v)eccentricity is the diameter diam (G). The circumference of a graph G with at least one cycle is the length of longest cycle in G, and is denoted by circum (G). As usual P_n , C_n and K_n are respectively the path, cycle and complete graphs of order n, $K_{r,s}$ is the complete bipartite graph with two partite sets containing r and s vertices. Any undefined term or notations in this paper can be found in [2].

2. PRELIMINARY NOTES
The line graph L(G) of G has the edges of G as its vertices which are adjacent in L(G) if and only if the corresponding edges are adjacent in G. We call the complement of line graph L(G) as the jump graph of G found in[4]. The jump graph J(G) of a graph G is the graph defined as E(G) and in which two vertices are adjacent if and only if they are not adjacent in G. Since both L(G) and J(G) are defined the edge set of a graph G. isolated vertices of G (if G has) play no role in line graph and jump graph transformation. Here we assume that the graph G under consideration is nonempty and has no isolated vertices found in [4].

Definition 2.1: We define the inverse line domination number of jump graph. Let G= (v, E) be a graph. Let D be a minimum line dominating set in a graph G. E-D contains a line dominating set D' of G then D' is called inverse line dominating set with respect to D, the minimum cardinality of an inverse line dominating set of a graph G is called the inverse line domination number of G and is denoted by $\gamma^{\prime -1}$(G).

Definition 2.2: Let J(G)= (V, E) be a jump graph. Let D b a minimum line dominating set in a jump graph J(G). if E-D contains a line dominating set D' of J(G),hen D' is called an inverse line dominating set with respect to D.

The minimum cardinality of an inverse line dominating set of a jump graph J(G) is called the inverse line domination number of J(G) and is denoted by $\gamma^{\prime -1}$(J(G)). For any graph G, with p≤ 4 the jump graph J(G)of G is disconnected. Since we study only the connected jump graph, we choose p>4. [5].

We recall following classical theorems to prove our results.

3. MAIN RESULTS
Theorem 3.1

(1) For any path P_n, with n≥5 $\gamma^{\prime -1}$(J(P_n)).=2

(2) For any cycle C_n with n≥5 $\gamma^{\prime -1}$(J(C_n)).=2

(3) For any complete graph K_p p≥ 5 $\gamma^{\prime -1}$(J(K_p)).=3

(4) For any complete bipartite graph $K_{r,s}$

$$\gamma^{\prime -1}(J(K_{r,s})).= \begin{cases} 2 \text{ for } K_{2,s} \text{ where } s>2 \\ 3 \text{ for } K_{m,n} \text{ where } m,,n \geq 3 \end{cases}$$

(5) For any wheel W_p

$$\gamma^{\prime -1}(J(W_p))..= \begin{cases} 3 \text{ for } p=5,6 \\ 2 \text{ for } p\geq 7 \end{cases}$$

Theorem 3.2 For any connected graph G,

$\gamma^{\prime -1}$(J(G)). ≥2

Proof of the theorem is obvious.

Theorem 3.3 For any connected graph G with diameter, diam(G) ≥ 2 γ^{r-1}(J(G)). ≥ 2.

Proof: Let uv be a path of maximum distance in G, then d(u, v)=diam(G) we can prove the theorem with the following cases.

Case(i) For diam(G) =2 choose a vertex v_1 of eccentricity 2 with maximum degree among others, let E-D= { e_1, e_2……..} corresponding the elements {v_1, v_2,…..} forming an inverse dominating set in jump graph J(G). Every vertex u∉ E –D is adjacent to a vertex E – D. Hence E – D is a minimum inverse line dominating set. So, the inverse line dominating number of the jump graph will e equal to the degree of v_1 also by theorem3.3 we say γ^{r-1}(J(G)). > 2.

Case(ii)For diam(G)>2. Let e_1 be any edge adjacent to u and e_2 be an edge adjacent to v.

Let { e_1, e_2 } ⊆ E(G) form a corresponding vertex set { v_1, v_2 } ⊆ V(J(G)). These two vertices form an inverse dominating set in jump graph. since there vertices { v_1, v_2 } are adjacent to all other vertices

of E – D(J(G)) it follows that { v_1, v_2 } becomes minimum inverse line dominating set.

Hence γ^{r-1}(J(G))=2.

In view of above cases we can conclude that for any connected graph G γ^{r-1}(J(G)). ≥2.

Theorem 3.4. For any tree T with diameter greater than 3 γ^{r-1}(J(G)). ≥ n.

Proof If the diameter is less than or equal to 3, then the jump graph will be disconnected.

Let u,v be a maximum length in a tree T where diameter is greater than 3. Let e_1 be the pendent edge adjacent to u and e_k be the pendent edge adjacent to v. The edge set e_i, i=1,2,3……n of J(T) corresponding to the vertices in T will form an inverse line dominating set of J(T). Since all the other edges of E(J(T)) are adjacent with e_i i=1,2,3,…….n it form a minimum inverse line dominating set

Hence γ^{r-1}(J(G)). = n.

Theorem 3.5. For any connected G(p,q) graph γ^{r-1}(J(G)).≤ p - ∆(G) where ∆(G) is the maximum degree of G.

Proof: Let E –D = { e_1, e_2, e_3……e_n } be the set of edges of G and E_1 = (E – D) – e_1 where e_1 is one of the edge with maximum degree. By definition of jump graph E(G) = V(JG)). Consider I = { v_1, v_2,….v_n} as the set of vertices adjacent to e_1 of G. Let H ⊆ E(JG)) e the set of edges of J(G) Hence H ⊆ E – I. Then H itself form a minimal inverse line dominating set.

Therefore γ^{r-1}(J(G)). ≤ |E| - |I| Hence γ^{r-1}(J(G)). ≤ p - ∆ (G).

4. CONCLUSION

Thus we conclude that inverse line domination number and the parameters of inverse line domination numbers of jump graphs like k_p, c_n, p_n, w_p and $k_{m,n}$

5. ACKNOWLEDGEMENTS

We would like to express our thanks to the referee, for their careful reading and valuable suggestions.

6. REFERENCES

[1] O. Ore Theory of Graphs. Amer. Math. Soci . Collq public 38 providence (1962)

[2] F. Harary Graph theory Adition-wesley Reading mass (1969)

[3] V.R. Kulli and S.C. Sigarkanti Inverse domination in Graph Nat. Acad Sci Lett 14 (1991) 473-475.

[4] G. Chatrand, H. Hevia, E.B Jarett, M. Schultz Subgraph distance in graph defined by edges transfers Discrete Math., 170 (1997) 63-79

[5] T. H Haynes, S.T. Hedetinemi and P. J. Slater Fundamentals of Domination in graphs Marcel Dekker Inc, New York (1998)

[6] G.S. Domke, J.E Dumbar and L.R Markus The inverse domination number of a graph Ars. Combin 72 (2004) 149-160.

[7] G. Chatrand and L. Lesniak, Graphs and Digraphs CRC (2004)

[8] P .D. Johnson , D.R. Pries and M.Walsh, On problem of Domke Haynes, Hedetinemi and markus Concerning the inverse domination number AKCEJ Graph Combin.72(2010) 217-222.

[9] A.Frendrup, M.A. Henning, b.Randerath and P. D. vestergoard On conjecture about inverse dominations in Graph Ars combinotorics 95A(2010) 103-111

[10] M. Karthikeyan, A Elumai Edge domination number of jump graphs. Int.. joumal of scientific& Engineering Research, vol6 issue3 (2015).pure and applied maths vol103 No.3 (2015) 477-483

[11] Y. B. Maralabhavi et.al., Domination number of jump graph Intn.Mathematical Forum vol8 (2013) 753-758

[12] S.R. Jayaram Line domination in graph Graphs and combinotorics vol3 (1987) 357-363.

[13] M. Karthikeyan, A. Elumalai Inverse domination number of jump graph. Int. joum.pure and applied maths vol103 No.3 (2015) 477-483 100-102.