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The Non-Split Domination Number of a Jump Graphs

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ABSTRACT
A dominating set D of a jump graph J(G) is a non split dominating set of a jump graph if the induced sub graph < E(J(G)) – D> is connected the non split domination $\surd_{ns}$ J(G) is minimum cardinality of a non-split dominating set. In this paper many bound of $\surd_{ns}$ J(G) are obtained and its exact values of some standard graphs are found. Also its relationship with other parameters are investigated.

Keywords
Graphs, domination number, Non split domination number

1. INTRODUCTION
The graph considered here are finite, undirected, non trivial and connected without loops and multiple edges.

A set D⊆ V(J(G)) is a dominating set of jump graph. If every vertex not in D is adjacent to a vertex D. The domination number of the jump graph J(G).

The dominating set D⊆ V(J(G)) is called a connected dominating set of a jump graph is the induced sub graph <D> is connected . The minimum cardinality of the connected dominating set in J(G) is its dominating number denoted by

$\surd_c$ (J(G)).

A dominating set D of a jump graph J(G)= (v(J(G) , E(JG)) is a split dominating set if the induced sub graph V(J(G)) – D disconnected. The split dominating number $\surd_s$ (J(G)) of J(G) is the minimum cardinality of a split dominating set.

The reader is referred to [1] [2] [3] and [4]for survey or results on domination.

Any undefined terms in this paper may be found in Harary[5] unless staed, the graph has p vertices and q edges.

The purpose of this paper is to introduce the concept of Non split domination.

A dominating set D of a graph J(G) is a nonsplit dominating set. If the induced sub graph v(J(G)) – D is connected. The non split domination number $\surd_{ns}$(J(G)) of J(G) is the minimum cardinality of a non split dominating set.

We call a set of vertices on √-setmif it is a dominating set with cardinality √(J(G)) similar a $\surd_c$ – set, a $\surd_s$- set and a $\surd_{ns}$-set are defined.

2. RESULTS
We start with some elementary results. Since their proofs are trivial we omit the same.

Theorem 2.1: For any graph G

√(J(G)) ≤ $\surd_{ns}$ (J(G))

Theorem 2.2. For any graph G

√(J(G)) = min { $\surd_s$(J(G)) , $\surd_{ns}$(J(G)) }

In [3] Cockayne and Hedetniemi gave necessary and sufficient conditions for a minimal dominating set.

Theorem A: A dominating set D of a graph G is minimal if and only if for each vertex v ∈ D one of the following condition is satisfy

(i) if there exists a vertex u ∈ V – D such that N[u] ∩ D = {v} and

(ii) v is an isolated vertex in < D >

Theorem 2.3: A non split dominating set D of J(G) is minimal if and only if for each vertex v in D one of the following conditions is satisfied.

(i) If there exists a vertex u ∈ V(J(G)) – D such that N[u] ∩ D={v}.

(ii) v is an isolated vertex in < D > and

(iii) N(v) ∩ (V(J(G)) – D) = ∅

Proof: Suppose D is minimal. On the contrary, if there exists a vertex v∈ D such that v does not satisfy any of the given conditions then by Theorem A D'= D – {v} is a dominating set of J(G) and by (iii) (v(J(G)) – D' is connected. This implies that D' is a non split dominating set of J(G), a contradiction This proves the necessity sufficiency is straight forward.

Next we observe a relationship between $\surd_{ns}$(J(G)) and $\surd_{ns}$(J(H)) where J(H) is any spanning sub graph of J(G) we omit the proof

Theorem 2.4. For any spanning sub graph J(H) of J(G)

$\surd_{ns}$(J(G)) ≤ $\surd_{ns}$(J(H)).

We obtain lower and upper bounds on $\surd_{ns}$(J(G)) respectively.

Theorem 2.5 For any graph J(G)

$\surd_{ns}$(J(G)) $\geq \frac{(2p-q-1)}{2}$

Proof**:** Let D be a $\surd_{ns}$(J(G))-set of J(G) Since

(v(J(G) – D) is connected.

q ≥ | ∇ (J(G)) – D | + | V(J(G)) – D | - 1

This proves the result..

Theorem 2.6. For any graph G

$\surd_{ns}$(J(G)) ≤ p – W(J(G)) is the Clique number of G.

Proof: Let S be a set of vertices of J(G) such that < S > is complete with |S|= w(j(g)) Then for any

u ∈ S (V(J(G)) – S) ∪ {u} is a non split dominating set of G.

Then the result holds.

Now we list the exact values of $\sqrt{}_{ns}(J(G))$ for some standard graphs..

Proposition 2.7.

(i) For any complete graph K_p with $p \geq 2$ vertices $\sqrt{}_{ns}(J(K_p)) = 1$.

(ii) For any complete bipartite graph $K_{m,n}$ with $2 \leq m \leq n$ $\sqrt{}_{ns}(J(K_{m,n})) = 2$

(iii) For any cycle c_p $\sqrt{}_{ns}(J(C_p)) = p-2$

(iv) For any wheel W_p $\sqrt{}_{ns}(J(W_p)) = 1$

(v) For any path P_p with $p \geq 3$ vertices $\sqrt{}_{ns}(J(P_p)) = p-2$

(vi) Our next result sharpens the inequality of Theorem 2.6 for tree.

Theorem 2.8. If **T** is a Tree which is not a star , then $\sqrt{}_{ns}(J(T)) \leq p-2$.

Proof: Since, t is not a star, there exists two adjacent cut vertices u and v with deg u, deg v ≥ 2 This implies that v $(J(G)) - \{u,v\}$ is a non split dominating set of J(T).

Thus the result holds.

Theorem 2.9. if $k(J(G)) \geq \beta_0(J(G))$ then

$\sqrt{}_{ns}(J(G)) = \sqrt{}(J(G))$ where k(J(G)) is the connectivity of jump graph J(G) and $\beta_0(J(G))$ is the independence number of J(G).

Proof: Let D be the $\sqrt{}$-set of jump graph J(G). Since $k(J(G)) > \beta_0(J(G)) \geq \sqrt{}(J(G))$. It implies that $< V(J(G)) - D >$ is connected. This proves that D is a $\sqrt{}_{ns}$-set of J(G).

Hence the result.

Theorem 2.10.: Let d be a $\sqrt{}_{ns}$-set of a connected graph G If no two vertices in $V(J(G)) - D$ are adjacent to a commom vertex in D then

$\sqrt{}_{ns}(J(G)) + \varepsilon(J(T)) \geq p$ where $\varepsilon(J(T))$ is the maximum number of end vertices in any spanning tree J(T) of J(G)..

Proof: Let D be a$\sqrt{}_{ns}$-set of J(G), given in the hypothesis. Since for any two vertices $u, v \in V(J(G)) - D$ there exists two vertices u_{1+} $v_1 \in D$ such that u_1 is adjacent to u but not v and v_1 is adjacent to a vertex v but not u_1 . This implies that there exists a spanning tree J(T) of $< V(J(G)) - D >$ in which each vertex of $V(G) - D$ is adjacent to a vertex of D.

This proved that $\varepsilon(J(T)) \geq |V(J(G)) - D|$.

Hence the result holds.

Theorem 2.11.: If $\delta(J(G)) + \omega(J(G)) \geq p+1$ then

$\sqrt{}_c(J(G)) + \sqrt{}_{ns}(J(G)) \leq p$ where

$.\delta(J(G))$ is the minimum degree of J(G).

Proof: By theorem 2.6 $\sqrt{}_{ns}(J(G)) \leq p - \omega(J(G)) + 1 \leq \delta(J(G))$

Let d be a $\sqrt{}_{ns}$- set of jump graph J(G). Then every ertex in D is adjacent to some vertex in $V(J(G)) - D$ Thus $<. V(J(G)) - D >$ is a connected dominating set of J(G). Since $< V(J(G)) - D >$ is connected .

Hence $\sqrt{}_c(J(G)) + \sqrt{}_{ns}(J(G)) \leq p$

Theorem 2.12: For any tree

$\sqrt{}_{ns}(J(T)) \geq p - m$. where m is the number of vertices adjacent to vertices.

Proof: If J(T) is K_2, the result is trivial. If J(T) has at least three vertices and D is a $\sqrt{}_{ns}$-set of J(T), then each vertex of $V(J(G)) - D$ is a cut vertices of J(T). Let S be the set of all cut vertices which are adjacent to end vertices with $|S|= m$ let $u \in V(J(G)) - D$. If $u \in S$ then $D = V(J(G)) - S$ and inequality holds. If $u \notin S$, then there exists a cut vrtex $v \in D$ adjacent to u. Further all vertices which are connected to v not through u also belonging to D This implies that $V(J(G)) - D$ has at most m vertices and inequality holds.

3. REFERENCES

[1] G. Chartrand and L.Lesnaik, *Graphsand Digraphs* chapman and hall Madras (1996*)*

[2] E.J. Cockayne, *Domination of undirected graphs.* A survey in theory and applications of GraphsLNM642 springer-verlog 1978 141-147.

[3] E.J. Cockayne and S.T. Hedetniemi a *Net work 7(1977)247-261.*

[4] 4. F. Harary *Graph theory* addition Wesley Reading mass (1969).

[5] V.R. Kulli and B. Janakiram The non split domination number of graph *Indian. J. phy.appl.Math.* 31(40 441-447 (2000)

[6] V. R. Kulli and B.Janakiram, The split domination number of a graph *Theory Notes of New York New York Academy of Sciences (1997) XXXII 16-19.*

[7] E. Sampath Kumar and H.B. Walikar J.Math.Phys.Sci 13 (1979)607-613.