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The Global Set-Domination Number in Jump Graphs

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ABSTRACT
Let J(G) be a co-connected jump graph. A set D ⊂ V(J(G)-D is a set dominating set (sd-set) if for every S ⊂ V(J(G)-D there exists a non empty set T ⊂ D such that the sub graph (S ∪ T) is connected. Further D is a global set dominating set, if D is an sd-set of both J(G) and J($\bar{G}$). The set domination number $\surd_s$ and the global set domination number $\surd_{sg}$ of J(G) are defined as expected

Keywords
Set domination,global set domination number

1. INTRODUCTION

Theorem 1
For the tree of order p with e end vertices $\surd_{sg}$(J(G)) = p-e

Theorem 2
If diam J(G) = 3 then $\surd_{sg}$J(G) ≤ $\surd_s$(J(G)) +2

If diam J(G)) = 4 then $\surd_{sg}$J(G) ≤ $\surd_s$(J(G)) + 1

If diam J(G) ≥ 5 then $\surd_{sg}$J(G) ≤ $\surd_s$(J(G))

Let J(G) = (V,E) be a jump graph. A set D ⊂ V(J(G) is a dominating set of J(G) if every vertex not in D is adjacent to some vertex in D. Further d is a global dominating set of J(G), if D is a dominating set of both J(G) and J($\bar{G}$) . The domination number √(J(G)) of J(G) is defined similarly the concept of global domination was first introduced by sampathkumar [4] and was also studied by Rall [3] Recently the concept of set domination for a connected graph was introduced by Sampath kumar and L. pushpa latha[5]. A set D⊂ V(J(G) is an set-dominating set (sd-set)of every set S ⊂ V(J(G))-D, there existd a non empty set T ⊂ D such that the sub graph < S ∪ T > induced by S ∪ Tis connected. The set-dominating number $\surd_s$J(G) = $\surd_{sg}$(J(G)) of jump graph J(G) is the minimum cardinality of an sd-set. Suppose J(G) is co-connected graph (i.e, both J(G) and J($\bar{G}$) are connected). The global set domination number $\surd_{sg}$=$\surd_{sg}$(J(G)) of J(G) is the minimum cardinality of an sd-set of both J(G) and J($\bar{G}$). The purpose of this paper is to initiate a study of $\surd_{sg}$.

Hence forth we consider only co-connected graph J(G). for a vertex v ∈ J(G). let N(v)= { u : uv ∈ E} and N[v] = N(v) ∪ {v}. Also

$\surd_s$ = $\surd_{sg}$(J(G)) .

Since every global sd-set is a global dominating set and $\surd_s \geq 2$ we have $2 \leq \surd_s \leq \surd_{sg}$ ….(1)

We observe that for a path p_n on n ≥4 vertices $\surd_{sg}(J(P_n))$ =n-2 and for a cycle c_n on n≥ 6 vertices $\surd_{sg}(J(C_n))$ = n-3 when $\surd_{sg}(J(C_5))$ = 3.

A $\surd_s$-set is minimum sd-set similarly we define $\surd_{sg}$-set etc., one can easily determine $\surd_{sg}$ for a tree.

Theorem 1. In a jump tree J(T) with p vertices and e end vertices that is not a star the set of non-end vertices form a minimum global sd-set and $\surd_{sg}$ J(T) =p-e.

Proof: It is known that the set d of all cut vertices of T form a $\surd_s$-set of T and $\surd_s$ = p – e [5] Clearly the sub graph V(J(T))-D in J($\bar{T}$) is complete. Since J(T) ≠ $K_{1,m}$ in J($\bar{T}$) each vertex in V(J(T)) – D is adjacent to some vertex in D this implies that D is an sd-set of J(T) also and $\surd_{sg}$ = p – e

We now determine some bounds for $\surd_{sg}$.

Theorem 2. Let J(G) be a co-connected sub graph of order p ≥ 4 then

$2 \leq \surd_{sg}(J(G)) \leq p - 2$ ………(2)

Proof: let u and v be adjacent vertices of degree at least two (such vertices clearly exist) Then V(J(G)) – {u, v} is a global sd-set of J(G) so $\surd_{sg}$(J(G)) ≤ p – 2.

The bounds in (2) are sharp. The upper bounds attained by paths of length at least 3 and the 5-cycle All jump graphs for which the lower bound is attained can be determined.

Theorem 3: For a jump graph J(G) of order p. $\surd_{sg}$=2 if and only if

diam J(G) = diam (J($\bar{T}$)) = 3 and either J(G) or J($\bar{T}$) has a bridge which is not an end edge.

Proof; Assume $\surd_{sg}$ = 2 since diam J(G) ≤ 3 and diam(J($\bar{G}$)) ≤ 3 Now, let D= {u,v} a $\surd_{sg}$-set of J(G) suppose u and v are adjacent in J(G).All vertices in V(J(G))-D are adjacent to either u or v (but not both). If all such vertices are adjacent to only u (or v) G and $\bar{G}$ is disconnected. Hence some vertices of V(J(G)- D are adjacent to u and some are adjacent to v. If all x ∈ N(v)- {u}, hen x and y are not adjacent in J(G),for otherwise

{u, v} will not be an sd-set in J(G). Thus uv is a bridge in J(G) that is not an end edge and d(x, y)=3=diam J(G) Also in J($\bar{G}$), d(u, v)=3 and hence diam J($\bar{G}$) = 3

Conversely, if J(G) has a bridge uv and is not an end edge and

diam J(G)= diam(J($\bar{G}$)) = 3, then every vertex in J(G)is adjacent to u or to v and hence {u, v} is a $\surd_s$-set in J(G). let N_G(u) be the set of all neighbors' of u in J(G), then N_G(u)= N_G[v] since uv is a bridge in J(G), every vertex of N_G(u) – {v} is adjacent to every vertex of N_G(u){u} in

J($\bar{G}$). Hence {u, v} is an sd-set of J($\bar{G}$) and √sg(J(G)) = 2.

Theorem 4; Let J(G) be a jump graph with cut vertices. Then

$\surd_{sg(}$J(G)) ≤$\surd_s$(J(G)) +1 = $\surd_c$(J(G))+1

Proof: We consider two cases

Case1. There exists a $\surd_s$-set D of J(G) all of whose vertices belong to a single block B of J(G).

Consider a vertex u∉ D such that u is a block $B_1 \neq$ B let D' =D∪ {u} we now show hat D' is an sd-set of J(G) let u, w ∈ V(J(G)). If v ,w belongs to a single block $B_i \neq B_1$ of J(G) then they are both adjacent to u in

$J(\bar{G})$. If u and w are in B_1 then in $J(\bar{G})$ both of them are adjacent to a vertex $u_1 \in D \cap (B - B_1)$ (note that $\surd_s \geq 2$) If v ∈ B_1 and w ∉ B_1 then

in $J(\bar{G})$ v is adjacent to u_1, and w is adjacent to u. Further the sub graph < { u, v,w,u_1}> is connected in J(G). This proves D' is an sd-set of J(G) and

$\surd_{sg}(J(G)) \leq |D'| \leq \surd_s(J(G)) + 1$.

Case 2. Case 1 is not true.

In this caswe for every $\surd_s$-set D of J(G) at least two vertices of D belong to different blocks of J(G) one can easily show tha d is also an sd-set of J $J(\bar{G})$. Hence $\surd_{sg}(J(G)) = \surd_s(J(G)(= \surd_c(J(G))$

Theorem 5. Le J(G) be a jump graph having diameter atleastfive and let D ⊂ V(J(G). Then D is a minimal sd-set of J(G) if and only if D is a minimal global sd-set of J(G).

Proof; Suppose D is minimal global sd-set of J(G). let u and v be such that d(u,v)≥5. Then D∩N[u] ≠ϕ and D∩N[v] ≠ϕ. Let u_1 ∈ D∩N[u] and

v_1 ∈ D∩N[v] . Since d(u, v) ≥ 5 u_1 and v_1 are non adjacent in J(G) and hence they are adjacent in J(G). Also no vertex in V(G)- { u_1, v_1 } is adjacent to both u_1 and v_1 in J(G) ssince otherwise d(u, v)┘≤ 5

Now in $J(\bar{G})$, each vertex is adjacent to u_1 or v_1(or both) and hence {u_1, v_1} is a connected dominating set of $J(\bar{G})$,Since every connected dominating set is an sd-set {u_1, v_1} is an sd-set of $J(\bar{G})$,. This proves that D is an minimal global sd-set of J(G).

Conversly, If D is a minimal global sd-set of J(G) and is noy a minimal sd-set of J(G), then there exists x∈ D such that D-{x} is also an sd-set of J(G). As before, if v_1 ∈ { D –{x}}∩ N[u] and v_1 ∈ { D –{x}}∩ N[v]

Then {u_1,v_1} is an sd-set of J(G) and hence D – {x] is a global sd-set of J(G) a contradiction Hence D is also a minimal sd-set of J(G).

2. CONCLUSION

In this paper we studied some characterization of some graphs by global set domination. It can used for further research work on set domination theory.

3. ACKNOWLEDGEMENT

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