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Perfect Line Domination in Graphs

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ABSTRACT
Here we are defining perfect line domination set and some results on perfect line domination.

Keywords
Line domination, perfect line domination, neighborhood, minimal line domination

1. INTRODUCTION
Definition 1. 1 : Perfect line dominating set:
A subset S of E(G) is said to be perfect line dominating set if for each edge not in S, e is adjacent to exactly one edge of S.

Consider the path P_4 with 4 vertices and edge are { e_1, e_2, e_3 }. The set { e_2 } is perfect line dominating set in this graph.

It may be noted that If G is a graph then E(G) is always a perfect line dominating set.

Definition 1.2: Minimal perfect line dominating set:

A perfect line dominating set S of the graph G is said to be minimal perfect line dominating set if each line e in S, S-{e} is not a perfect line dominating set.

It may be noted that it is not necessary that a proper subset of minimal perfect line dominating set is not a perfect line dominating set

Example 1. 3. Consider the cycle graph G $=C_5$ with 5 vertices. Then obviously E(G) is a minimal perfect line dominating set of G.

However the set { e_1, e_2 } is proper subset of E(G) and is a perfect line dominating set in the graph G.

Definition 1.4: Minimum perfect line dominating set:

A perfect line dominating set with smallest cardinality is called set of the graph minimum perfect line dominating set it is called γ'_{pf}

Definition1.5: Perfect line domination number:

The cardinality of a minimum perfect line dominating set is called the perfect domination number of the graph G. it is denoted by γ'_{pf}(G).

The perfect line domination number of cycle C_6 is 2, C_5 is 2 and that of path P_4 is 1.

Definition 1.6: perfect private line neighborhood:

Let S be the subset of E (G) and eϵ S then the perfect private neighborhood of e with respect to S = P_{pf} [e, S] = { x ∈ E(G)-S ; N(x) ∩ { e } } ∪ { e, if e is adjacent to no line of S or at least line of S.

Theorem 1.7: a perfect line dominating set S of G is minimal perfect line dominating set if and only if for each line e in S P_{pf} [e, S] is nonempty.

Suppose S is minimal and eϵ S . Therefore there is a line x not in S- {e} or x is adjacent to at least two lines of S –{e}.

If x=e then this implies that eϵ P_{pf} [e, S]

If x≠ e then it is impossible that x is adjacent to at least two lines of S – {e}. because S is a perfect line dominating set. Therefore x is not adjacent to any line of S – {e}. Since S is a perfect dominating set x is adjacent to only e in S. That is N(x)∩S ={ e } then x ϵ P_{pf} [e, S].

Conversely, suppose eϵ S and P_{pf} [e, S] contains some line x of G.

If x=e then x is either adjacent to at least two lines of S –{ e} Then S –{ e} is not perfect line dominating set

If e≠e then N(x)∩ S = { e] implies that x is not adjacent to any line of S –{ e}.

Thus in all cases S – { e} is not a perfect line dominating set, if eϵ S. Thus S is minimal.

Example 1.8. Consider the path G= P_5 with five lines e_1, e_2, e_3, e_4 e_5. Note that S ={ e_2, e_4} is minimum and therefore minimal perfect line dominating set P_{pf} {e_2, S} = {e_1, e_2}

We define the following symbols,

E^+_{pf}= { e ∈ E(G) ; γ'_{pf} (G) ≤ γ'_{pf} (G-e)}

E^-_{pf} ={ e ∈ E(G) ; γ'_{pf} (G) > γ'_{pf} (G-e)}

E^0_{pf} = { e ∈ E(G) ; γ'_{pf} (G) = γ'_{pf} (G-e)}

Remark; The above sets are mutually disjoint and their union is E(G)

Lemma; **1. 9.** Let e ∈ E(G) and suppose e is a pendent line and has a neighbor x if e ∈ E^-_{pf} then

γ'_{pf} (G-e) = γ'_{pf} (G) – 1.

Let S_1 be a minimum line dominating set of

E – { e} if x∈ S_1 then S_1 is a perfect line dominating set of G with | S_1 | < γ'_{pf} (G)

i.e., γ'_{pf} (G) ≤ | S_1 | ≤ γ'_{pf} (G-e) this is a contradiction, therefore x∉ S_1 let S = S_1 ∪ { x }. Then S is minimum perfect line dominating set of G. Therefore γ'_{pf} (G) = | S | = |S_1 | + 1 = γ'_{pf} (G – e) + 1

Hence the lemma.

Theorem 1.10. Let e e a line of G Then e ∈ E^+_{pf} if and only if the following conditions are satisfies, i) e belongs to every γ'_{pf} set of G, (ii) no subset s of G – { e] which is either disjoint from N[e] or intersects N[e] in at least two linesand |S| ≤ γ'_{pf} can be perfectly dominating set of

G – {e}.

Proof: (i) Suppose $e \in E^{+}_{pf}$,

Suppose S is a γ'_{pf} set of G which does not contains e then S is a perfect dominating set of

G – {e}.
Therefore γ'_{pf} (G-e) ≤ |S| = γ'_{pf} (G). thus $e \notin E^{+}_{pf}$. This is a contradiction. Then e must belong to every γ'_{pf} set of G,

(ii) If there is set S which satisfies the condition stated in (ii) then S is a perfect line dominating set of G – {e] and therefore γ'_{pf} (G-e) ≤ γ'_{pf} (G) This is a contradiction.

Conversely, assume that (i) and (ii) hold

Suppose $e \in E^{0}_{pf}$. Let s be a minimum perfect line dominating set of G – { e }.

Then |S| = γ'_{pf} (G).

Suppose e is not adjacent to any line of S. Then S is disjoin from N[e]. |S| = γ'_{pf} (G) . and S is perfectly line dominating set of G – {e}. This violates (ii)

Suppose e is adjacent to exactly one line of S then S is minimum perfect line dominating set of G not containing e which violates (i)

Suppose e is adjacent to at least two lines of S, then |S| ∩ N[e] in at least two lines and S is perfectly line dominating set of G – {e] with |S| = γ'_{pf} (G) which again violates (ii).

Thus $e \in E^{0}_{pf}$ implies (i) and (ii) violated.

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