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# Batch Arrival Retrial Queue with Negative Customers, Multi-optional Service and Feedback

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## ABSTRACT
This paper deals with single server retrial queueing model where in addition to regular arriving customers there are negative arrivals. Server provides M types of service. Positive customers arrive in batches according to Poisson process. If the server is idle upon the arrival of a batch, one of the customers in the batch receives any one of the types immediately and the rest join the orbit. Otherwise, the arriving batch joins the orbit. After completion of service, the customer may join the orbit as a feedback customer or depart the system. Negative arrival has the effect of removing the customer in service from the system. Performance analyses are obtained using supplementary variable technique. Stochastic decomposition law is verified and numerical results are also discussed.

## Keywords
Retrial queue - negative customers -multi-optional service-feedback

## 1. INTRODUCTION
The phenomenon of feedback in the retrial queueing system occurs in many practical situations. Many authors including Krishnakumar et al. [3], Mokaddis et al. [6] and Lee and Jang [4] analysed retrial queueing systems with feedback. Ramanath and Lakshmi [8] studied M/G/1 retrial queue with second multi-optional service and feedback. Recently, Baruah et al.[1] and Rajadurai et al.[7] analyzed a batch arrival retrial queueing system with two types of service, where server provides optional re-service.

Queues with negative arrivals called G-queues were first introduced by Gelenbe [2] with a view to modeling neural networks. In recent years, a variety of industrial applications have created interest in the modeling of reliability in G-queues. Liu et al.[5] analysed an M/G/1 retrial G-queue with server breakdown and feedback.

In this paper, we extend Baruah et al.[1] to retrial G-queue with multi-optional essential service and feedback.

## 2. MODEL DESCRIPTION
Consider a single server retrial queueing system with two types of independent arrivals, positive and negative. Positive customers arrive in batches according to Poisson process with rate $\lambda^+$. The batch size Y is a random variable with distribution function $P(Y=k)=C_k$, $k=1,2,..,\infty$ and probability generating function $C(z)=\sum_{k=1}^{\infty} c_k z^k$ having first two moments $m_1$ and $m_2$.

The server provides M types of services. Customers opt the $i^{th}$ type of service with probability $p_i$ $(1 \le i \le M)$ $(p_1+p_2+\ldots+p_M = 1)$. If the arriving batch of positive customers finds the server free, one of the arrivals receives service immediately. Otherwise the arriving batch joins the retrial queue. The retrial time is generally distributed with distribution function A(x), density function a(x), Laplace

Stieltje's transform $A^*(\theta)$ and conditional completion rate $\eta(x) = a(x)/[1-A(x)]$.

The service time of type $i(i=1,2,\ldots,M)$ follows a general distribution with distribution function $B_i(x)$, density function $b_i(x)$, Laplace Stieltje's transform $B_i^*(\theta)$, $n^{th}$ factorial moments $\mu_i^{(n)}$ and conditional completion rate $\mu_i(x) = b_i(x)/[1-B_i(x)]$, for $(i=1,2,\ldots,M)$. After receiving service, the customer may join the orbit as a feedback with probability $\delta$ or depart the system with its complementary probability $1-\delta$.

Negative customers arrive singly according to Poisson process with rate $\lambda^-$. The arrival of a negative customer removes the positive customer being in service from the system and makes the server down. When the server fails, it stops providing service and is sent for repair immediately. The repair time also follows general distribution with distribution function $R_i(x)$, density function $r_i(x)$, Laplace Stieltje's transform $R_i^*(\theta)$, $n^{th}$ factorial moments $\beta_i^{(n)}$ and conditional completion rate $\beta_i(x) = r_i(x)/[1-R_i(x)]$, for $(i=1,2,\ldots,M)$. Various stochastic process involved in the system are independent of each other.

## 3. ANALYSIS OF THE STEADY STATE DISTRBUTION
The state of the system at time t can be described by the Markov process $\{X(t),t\ge 0\} = \{C(t), N(t), \xi(t), t\ge 0\}$ where C(t) denotes the server state 0,i or i+M according as the server being idle, busy or under repair. N(t) corresponds to the number of customers in the orbit. If C(t)=0, then $\xi(t)$ represents the elapsed retrial time. If C(t) = i , then $\xi(t)$ represents the elapsed service time. If C(t)=i+M, then $\xi(t)$ represents the elapsed repair time of the failed server $(1 \le i \le M)$.

## 4. STEADY STATE DISTRBUTION
For the process $\{C(t), t\ge 0\}$, define the probabilities

$$I_0(t) = P\{C(t)=0,N(t) = 0\}$$

$$I_n(x,t)dx = P\{C(t)=0,N(t) = n,\ x < \xi(t) \le x+dx\},\ n \ge 1$$

$$P_n^{(i)}(x,t)dx = P\{C(t)=i,N(t)=n,x< \xi(t) \le x+dx\},\ n \ge 0,$$
$i=1,2,\ldots,M$

$R_n^{(i)}(x,t)dx = P\{C(t)=i+M, N(t)=n, x<\xi(t)\le x+dx\},\ n\ge 0,$

$i=1,2,\ldots,M$

The governing equations of the model under study in equilibrium state are

$$\lambda^+ I_0 = \sum_{i=1}^{M}\left[\bar{\delta}\int_0^\infty P_0^{(i)}(x)\mu_i(x)dx + \int_0^\infty R_0^{(i)}(x)\beta_i(x)dx\right] \tag{1}$$

$$\frac{\partial}{\partial x}I_n(x) = -(\lambda^+ + \eta(x))I_n(x),\ n\ge 1 \tag{2}$$

$$\frac{\partial}{\partial x}P_n^{(i)}(x) = -(\lambda+\mu_i(x))P_n^{(i)}(x) + (1-\delta_{0n})\sum_{k=1}^{n}\lambda^+ c_k P_{n-k}^{(i)}(x),\quad n\ge 0, i=1,2,\ldots,M \tag{3}$$

$$\frac{\partial}{\partial x}R_n^{(i)}(x) = -(\lambda^+ + \beta_i(x))R_n^{(i)}(x) + (1-\delta_{0n})\sum_{k=1}^{n}\lambda^+ c_k R_{n-k}^{(i)}(x),\quad n\ge 0, i=1,2,\ldots,M \tag{4}$$

The boundary conditions are

$$I_n(0) = \sum_{i=1}^{M}\left[\delta\int_0^\infty P_{n-1}^{(i)}(x)\mu_i(x)dx + \bar{\delta}\int_0^\infty P_n^{(i)}(x)\mu_i(x)dx\right] + \sum_{i=1}^{M}\int_0^\infty R_n^{(i)}(x)\beta_i(x)dx,\quad n\ge 1 \tag{5}$$

$$P_0^{(i)}(0) = p_i\left[\lambda^+ c_1 I_0 + \int_0^\infty I_1(x)\eta(x)dx\right],\ i=1,2,\ldots,M \tag{6}$$

$$P_n^{(i)}(0) = p_i\left[\lambda^+ c_{n+1}I_0 + \int_0^\infty I_{n+1}(x)\eta(x)dx + \sum_{k=1}^{n}\lambda^+ c_k\int_0^\infty I_{n-k+1}(x)dx\right],\quad n\ge 1, i=1,2,\ldots,M \tag{7}$$

$$R_n^{(i)}(0) = \lambda^-\left[\int_0^\infty P_n^{(i)}(x)dx\right],\ n\ge 0,\ i=1,2,\ldots,M \tag{8}$$

The normalizing condition is

$$I_0 + \sum_{n=1}^{\infty}\int_0^\infty I_n(x)dx + \sum_{n=0}^{\infty}\sum_{i=1}^{M}\left[\int_0^\infty P_n^{(i)}(x)dx + \int_0^\infty R_n^{(i)}(x)dx\right] = 1 \tag{9}$$

Define the probability generating functions

$I(x,z) = \sum_{n=1}^{\infty} I_n(x)z^n$, $P^{(i)}(x,z) = \sum_{n=0}^{\infty} P_n^{(i)}(x)z^n$ and $R^{(i)}(x,z) = \sum_{n=0}^{\infty} R_n^{(i)}(x)z^n$, $i=1,2,\ldots,M$.

Multiplying equations (2) – (8) by $z^n$ and summing over all possible values of n, we obtain the following results:

$$\left[\frac{\partial}{\partial x} + (\lambda^+ + \eta(x))\right]I(x,z) = 0 \tag{10}$$

$$\left[\frac{\partial}{\partial x} + (\lambda^+ + \lambda^- - \lambda^+ C(z) + \mu_i(x))\right]P^{(i)}(x,z) = 0,\quad i=1,2,\ldots,M \tag{11}$$

$$\left[\frac{\partial}{\partial x} + (\lambda^+ - \lambda^+ C(z) + \beta_i(x))\right]R^{(i)}(x,z) = 0,\ i=1,2,\ldots,M \tag{12}$$

$I(0,z) =$

$$\sum_{i=1}^{M}\left[z\delta\int_0^\infty P^{(i)}(x,z)\mu_i(x)dx + \bar{\delta}\int_0^\infty P^{(i)}(x,z)\mu_i(x)dx + \int_0^\infty R^{(i)}(x,z)\beta_i(x)dx\right] - \lambda^+ I_0 \tag{13}$$

$$P^{(i)}(0,z) = \frac{p_i}{z}\left[\int_0^\infty I(x,z)\eta(x)dx + \lambda^+ C(z)\left[\int_0^\infty I(x,z)dx + I_0\right]\right] \tag{14}$$

$$R^{(i)}(0,z) = \lambda^-\int_0^\infty P^{(i)}(x,z)dx,\ i=1,2,\ldots,M \tag{15}$$

Solving the partial differential equation (10) - (12), we get

$$I(x,z) = I(0,z)e^{-\lambda^+ x}[1-A(x)] \tag{16}$$

$$P^{(i)}(x,z) = P^{(i)}(0,z)e^{-(\lambda^+ + \lambda^- - \lambda^+ C(z))x}[1-B_i(x)],\quad i=1,2,\ldots,M \tag{17}$$

$$R^{(i)}(x,z) = R^{(i)}(0,z)e^{-(\lambda^+ - \lambda^+ C(z))x}[1-R_i(x)],\quad i=1,2,\ldots,M \tag{18}$$

Using equation (16) in equation (14), we get

$$P^{(i)}(0,z) = \frac{p_i}{z}\left[\lambda^+ I_0 C(z) + I(0,z)\left[C(z) + (1-C(z))A^*(\lambda^+)\right]\right],\quad i=1,2,\ldots,M \tag{19}$$

Using equation (17) in equation (15), we obtain

$$R^{(i)}(0,z) = \lambda^- P^{(i)}(0,z)\bar{B}_i^*(g(z)), \tag{20}$$

where $g(z) = \lambda^+ + \lambda^- - \lambda^+ C(z)$

Using equations (17),(18),(19) and (20) in equation (13) and on solving we get

$$I(0,z) = \frac{\lambda^+ I_0[C(z)\sum_{i=1}^{M}p_i([\delta(z-1)+1]B_i^*(g(z))g(z) + \lambda^-(1-B_i^*(g(z)))R_i^*(h(z))) - zg(z)]}{D(z)} \tag{21}$$

where,

$D(z) = zg(z) - ([A^*(\lambda^+) + C(z)(1-A^*(\lambda^+))]\bullet \sum_{i=1}^{M}p_i([\delta(z-1)+1]B_i^*(g(z))g(z) + \lambda^-(1-B_i^*(g(z)))R_i^*(h(z))))$

$h(z) = \lambda^+ - \lambda^+ C(z)$

Using equation (21), the equation (19) becomes

$$P^{(i)}(0,z) = \lambda^+ I_0 A^*(\lambda^+)p_i[C(z)-1]/D(z) \tag{22}$$

Using equation (22) in equation (20), we get

$R^{(i)}(0,z) =$

$$\lambda^+\lambda^- I_0 A^*(\lambda^+)p_i[C(z)-1][1-B_i^*(g(z))]/D(z) \tag{23}$$

The partial probability generating function of the orbit size when the server is idle is given by

$I(z) = \int_0^\infty I(x,z)dx =$

$$(I_0(1-A^*(\lambda^+))[C(z)\sum_{i=1}^{M}p_i([\delta(z-1)+1]B_i^*(g(z))g(z) + \lambda^-(1-B_i^*(g(z)))R_i^*(h(z))) - zg(z)])/D(z) \tag{24}$$

The partial probability generating function of the orbit size when the server is busy in type i service is given by

$$P^{(i)}(z) = \int_0^\infty P^{(i)}(x,z)dx =$$

$$\lambda^+ I_0 A^*(\lambda^+) p_i (C(z)-1)[1-B_i^*(g(z))]/D(z), i = 1,2,...,M \quad (25)$$

The partial probability generating function of the orbit size when the server is under repair is given by

$$R^{(i)}(z) = \sum_{i=1}^{M} \int_0^\infty R^{(i)}(x,z)dx =$$

$$-\lambda^- I_0 A^*(\lambda^+) p_i [1-B_i^*(g(z))][1-R_i^*(h(z))]/D(z) \quad (26)$$

The unknown constant $I_0$ can be obtained by using the normalizing condition (9) as

$$I_0 = \frac{\lambda^-[1-m_1(1-A^*(\lambda^+))-\lambda^+ m_1[1-\sum_{i=1}^{M} p_i B_i^*(\lambda^-)] -\lambda^-\delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-) - \lambda^+\lambda^- m_1 \sum_{i=1}^{M} p_i \beta_i^{(1)})(1-B_i^*(\lambda^-))}{\lambda^- A^*(\lambda^+)[1-\delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-)]} \quad (27)$$

**Probability that the server is idle in the non-empty system is given by**

$$I = \lim_{z\to 1} I(z) = \frac{(1-A^*(\lambda^+))\{\lambda^-[m_1 - 1 + \delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-)] + \lambda^+ m_1[1+\sum_{i=1}^{M} p_i[-B_i^*(\lambda^-)+\lambda^-\beta_i^{(1)}(1-B_i^*(\lambda^-))\}}{\lambda^- A^*(\lambda^+)[1-\delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-)]} \quad (28)$$

**Probability that the server is busy in type i service is given by**

$$P = \lim_{z\to 1} \sum_{i=1}^{M} P^{(i)}(z) = \frac{\sum_{i=1}^{M} \lambda^+ m_1 p_i (1-B_i^*(\lambda^-))}{\lambda^-[1-\delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-)]} \quad (29)$$

**Probability that the server is under repair is given by**

$$R = \lim_{z\to 1} \sum_{i=1}^{M} R^{(i)}(z) = \frac{\lambda^+ m_1 \sum_{i=1}^{M} \beta_i^{(1)} p_i (1-B_i^*(\lambda^-))}{[1-\delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-)]} \quad (30)$$

**The probability generating function of the orbit size is**

$$P_q(z) = I_0 A^*(\lambda^+) g(z)[z-1][1-\delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-)]/D(z) \quad (31)$$

**The probability generating function of the system size is**

$$P_s(z) = I_0 A^*(\lambda^+)[z-1][\lambda^- + (h(z)-\delta g(z))\sum_{i=1}^{M} p_i B_i^*(\lambda^-)]/D(z) \quad (32)$$

# 5. PERFORMANCE MEASURES

**Mean number of customers in the orbit is derived as**

$$L_q = P_q'(1) = \frac{Nr''Dr' - Nr'Dr''}{2Dr'^2} \quad (33)$$

where Nr(z) and Dr(z) are the numerator and denominator of $P_q(z)$

$$Nr' = I_0\lambda^-[1-\delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-)]$$

$$Nr'' = -2I_0\lambda^+ m_1[1+\lambda^-\delta\sum_{i=1}^{M} p_i \mu_i^{(1)} - \delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-)]$$

$$Dr' = \lambda^-[1-m_1(1-A^*(\lambda^+))-\lambda^+ m_1[1-\sum_{i=1}^{M} p_i B_i^*(\lambda^-)] -\lambda^-\delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-) - \lambda^+\lambda^- m_1\sum_{i=1}^{M} p_i\beta_i^{(1)})(1-B_i^*(\lambda^-))$$

$$\begin{aligned} Dr'' = &-2\lambda^+ m_1 - \lambda^+ m_2 - \lambda^- m_2(1-A^*(\lambda^+)) \\ &-\sum_{i=1}^{M} p_i[2\lambda^+\lambda^- m_1\delta\mu_i^{(1)} - 2\lambda^+ m_1\delta B_i^*(\lambda^-) - 2\lambda^{+2}\lambda^- m_1^2\mu_i^{(1)}\beta_i^{(1)} \\ &+\lambda^-(1-B_i^*(\lambda^-))[\lambda^{+2} m_1^2\beta_i^{(2)} + \lambda^+ m_2\beta_i^{(1)}] - 2\lambda^{+2} m_1^2\mu_i^{(1)} \\ &-\lambda^+ m_2 B_i^*(\lambda^-)] - 2m_1(1-A^*(\lambda^+))\sum_{i=1}^{M} p_i[(\lambda^-\delta B_i^*(\lambda^-) \\ &-\lambda^+ m_1 B_i^*(\lambda^-) + \lambda^+\lambda^- m_1\beta_i^{(1)}(1-B_i^*(\lambda^-))] \end{aligned}$$

**Mean number of customers in the system is**

$$L_s = P_s'(1) = \frac{Nr_1''Dr' - Nr_1'Dr''}{2Dr'^2} \quad (34)$$

where $Nr_1(z)$ and Dr(z) denotes the numerator and denominator of $P_s(z)$

$$Nr_1' = I_0\lambda^-[1-\delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-)]$$

$$Nr_1'' = -2I_0\lambda^+ m_1[\lambda^-\delta\sum_{i=1}^{M} p_i\mu_i^{(1)} - (\delta-1)\sum_{i=1}^{M} p_i B_i^*(\lambda^-)]$$

# 6. RELIABILITY INDICES

The system availability A(t) at time t is the probability that the server is either working for a customer or in an idle period. Then under steady state condition availability of the server is shown to be

$$A = 1 - \sum_{i=1}^{M} R^{(i)} = 1 - \frac{\lambda^+ m_1\sum_{i=1}^{M}\beta_i^{(1)} p_i(1-B_i^*(\lambda^-))}{[1-\delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-)]} \quad (35)$$

Steady state failure frequency of the server is

$$F = \lambda^-\sum_{i=1}^{M} P^{(i)} = \frac{\lambda^+ m_1\sum_{i=1}^{M} p_i(1-B_i^*(\lambda^-))}{[1-\delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-)]} \quad (36)$$

# 7. STOCHASTIC DECOMPOSITION

**Theorem:** The decomposition property states that the number of customers in the system in steady state at a random point of time ($L_s$) is distributed as the sum of two independent random variables, one of which is the number of customers in the corresponding standard queueing system in steady state at a random point of time L and the other random variable may have different probabilistic interpretation in specific cases depending on the vacation scheduled ($L_I$).

**Proof:**

The probability generating function π(z) of the system size in the classical batch arrival G-queue and unreliable server with feedback. Then equation (32) is given by

$\pi(z)=$

$$\frac{\begin{array}{l}[z-1][\lambda^- + (h(z)-\delta g(z))\sum_{i=1}^{M} p_i B_i^*(\lambda^-)][\lambda^- - \lambda^+ m_1[1-\sum_{i=1}^{M} p_i B_i^*(\lambda^-)] \\ -\lambda^-\delta \sum_{i=1}^{M} p_i B_i^*(\lambda^-) - \lambda^+\lambda^- m_1 \sum_{i=1}^{M} p_i \beta_i^{(1)})(1-B_i^*(\lambda^-))]\end{array}}{\lambda^-[1-\delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-)][zg(z)-\sum_{i=1}^{M} p_i([\delta(z-1)+1]B_i^*(g(z))g(z)+\lambda^-(1-B_i{}^*(g(z)))R_i{}^*(h(z)))]} \tag{37}$$

The probability generating function $\chi(z)$ of the number of customers in the orbit when the system is idle is given by

$$\chi(z) = \frac{I_0 + I(z)}{I_0 + I(1)}$$

$$= \frac{\left[\left[zg(z)-\sum_{i=1}^{M} p_i([\delta(z-1)+1]B_i^*(g(z))g(z)+\lambda^-(1-B_i{}^*(g(z)))R_i{}^*(h(z)))\right]\bullet\left[\begin{array}{l}\lambda^-[1-m_1(1-A^*(\lambda^+))]-\lambda^+ m_1[1-\sum_{i=1}^{M} p_i B_i^*(\lambda^-)]-\lambda^-\delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-) \\ -\lambda^+\lambda^- m_1 \sum_{i=1}^{M} p_i \beta_i^{(1)})(1-B_i^*(\lambda^-))\end{array}\right]\right]}{D(z)\left[\begin{array}{l}\lambda^- - \lambda^+ m_1[1-\sum_{i=1}^{M} p_i B_i^*(\lambda^-)]-\lambda^-\delta\sum_{i=1}^{M} p_i B_i^*(\lambda^-) \\ -\lambda^+\lambda^- m_1 \sum_{i=1}^{M} p_i \beta_i^{(1)})(1-B_i^*(\lambda^-))\end{array}\right]} \tag{38}$$

From equations (32),(37) and (38), it is observed that probability generating function of the number of customers in the system $P_s(z)$ is decomposed as $P_s(z) = \pi(z)\,\chi(z)$.

Hence, $L_s = L + L_I$.

## 8. NUMERICAL RESULTS

Numerical results are calculated by assuming the distributions of retrial times, service time and repair time as exponential with rates $\eta$,$\mu$ and $\beta$.

For the parameters $\lambda^+ = 0.2$, $\lambda^- = 0.2$ ,M=3, $p_1$=0.3, $p_2$=0.3, $p_3$=0.4, $m_1$=1.5, $m_2$=1,$\delta$=0.2,$\eta$=0.6,$\mu_1$=5, $\mu_2$=8, $\mu_3$=30, $\beta_1$=2, $\beta_2$=4, $\beta_3$=6, the performance measures $I_0$,I,P,R,*A* and *F* are calculated and presented in tables 1 to 4 respectively for various rates of $\lambda^+$, $\lambda^-$,$\delta$ and $\eta$.

Table 1 reveals that $I_0$ and *A* monotonically decrease and I,P,R and *F* increase as $\lambda^+$ increases. Table 2 indicates that increase in $\lambda^-$ increases $I_0$, R and *F* and decreases other performance measures. From Table 3 we observe that the parameters $\eta$ has no effect on the performance measures P,R,*A* and *F*. As $\eta$ increases $I_0$ increases and I decreases. Table 4 depicts the effect of $\delta$ on the performance measures. It is noted that I, P,R and *F* increase and $I_0$ and *A* decrease for increasing values of $\delta$.

Figures 1 to 4 reveal the trend of Lq against the pair i - ( $\lambda^+$, $\mu_1$), ii - ($\lambda^-$ ,$\mu_2$) , iii - ($\lambda^-$,$\beta_1$ ) and iv - ($\delta$, $\mu_1$).

**Table. 1 Performance measures by varying $\lambda^+$**

| $\lambda^+$ | $I_0$ | I | P | R | *A* | *F* |
|---|---|---|---|---|---|---|
| 0.1 | 0.8307 | 0.1478 | 0.0201 | 0.0015 | 0.9985 | 0.0040 |
| 0.2 | 0.6542 | 0.3027 | 0.0401 | 0.0030 | 0.9970 | 0.0080 |
| 0.3 | 0.4705 | 0.4648 | 0.0602 | 0.0045 | 0.9955 | 0.0120 |
| 0.4 | 0.2797 | 0.6341 | 0.0802 | 0.0060 | 0.9940 | 0.0160 |
| 0.5 | 0.0816 | 0.8106 | 0.1003 | 0.0075 | 0.9925 | 0.0201 |

**Table 2 Performance measures by varying $\lambda^-$**

| $\lambda^-$ | $I_0$ | I | P | R | *A* | *F* |
|---|---|---|---|---|---|---|
| 0.3 | 0.6548 | 0.3013 | 0.0394 | 0.0044 | 0.9956 | 0.0118 |
| 0.5 | 0.6560 | 0.2988 | 0.0381 | 0.0071 | 0.9929 | 0.0191 |
| 0.7 | 0.6571 | 0.2964 | 0.0369 | 0.0096 | 0.9904 | 0.0258 |
| 0.9 | 0.6582 | 0.2941 | 0.0358 | 0.0119 | 0.9881 | 0.0322 |
| 0.1 | 0.6592 | 0.2920 | 0.0347 | 0.0141 | 0.9859 | 0.0382 |

**Table 3 Performance measures by varying $\eta$**

| $\eta$ | $I_0$ | I | P | R | *A* | *F* |
|---|---|---|---|---|---|---|
| 0.2 | 0.0488 | 0.9081 | 0.0401 | 0.0030 | 0.9970 | 0.0080 |
| 0.4 | 0.5029 | 0.4540 | 0.0401 | 0.0030 | 0.9970 | 0.0080 |
| 0.6 | 0.6542 | 0.3027 | 0.0401 | 0.0030 | 0.9970 | 0.0080 |
| 0.8 | 0.7299 | 0.2270 | 0.0401 | 0.0030 | 0.9970 | 0.0080 |
| 1.0 | 0.7753 | 0.1816 | 0.0401 | 0.0030 | 0.9970 | 0.0080 |

**Table 4 Performance measures by varying $\delta$**

| $\delta$ | $I_0$ | I | P | R | *A* | *F* |
|---|---|---|---|---|---|---|
| 0.1 | 0.7249 | 0.2337 | 0.0358 | 0.0027 | 0.9973 | 0.0072 |
| 0.2 | 0.6542 | 0.3027 | 0.0401 | 0.0030 | 0.9970 | 0.0080 |
| 0.3 | 0.5601 | 0.3908 | 0.0457 | 0.0034 | 0.9966 | 0.0091 |
| 0.4 | 0.4358 | 0.5072 | 0.0530 | 0.0040 | 0.9960 | 0.0106 |
| 0.5 | 0.2639 | 0.6682 | 0.0632 | 0.0047 | 0.9953 | 0.0126 |

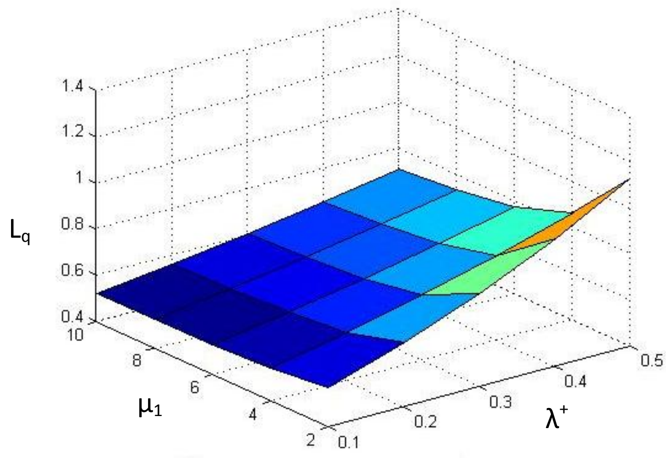


**Figure 1- $L_q$ versus $\lambda^+$ and $\mu_1$**

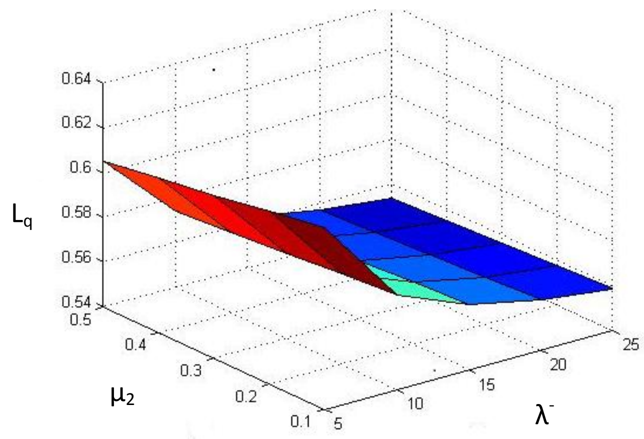


**Figure 2- $L_q$ versus $\lambda^-$ and $\mu_2$**

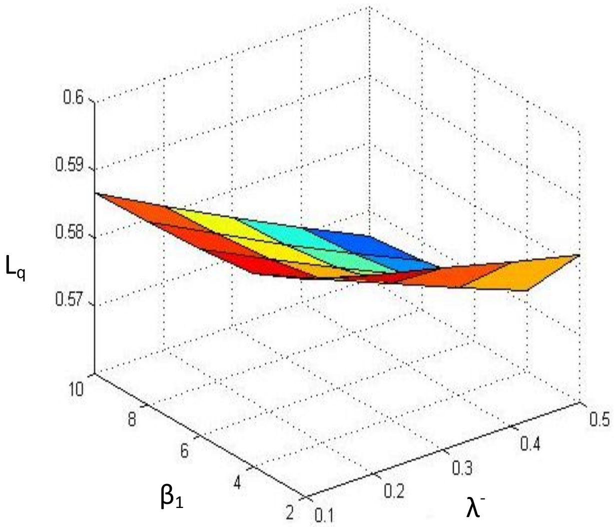


**Figure 3- $L_q$ versus $\lambda^-$ and $\beta_1$**

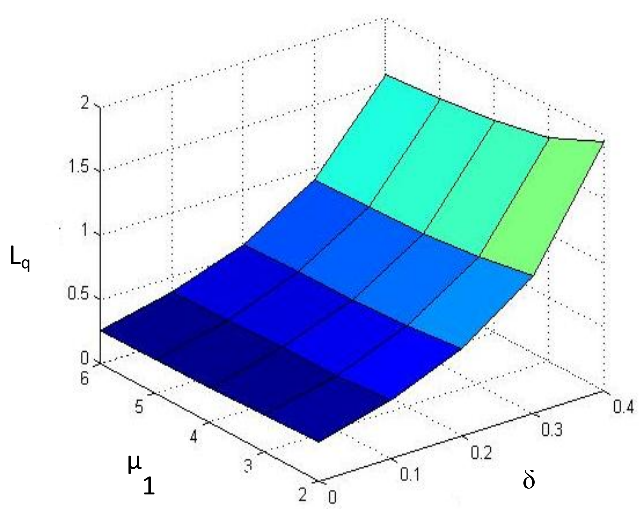


**Figure 4- $L_q$ versus $\delta$ and $\mu_1$**

# 9. CONCLUSIONS

In the present study, a single server batch arrival retrial queue with negative customers, multi-types of heterogenous service with feedback is analyzed. Analytical expressions for various performance measures are derived using supplementary variable technique. The stochastic decomposition law is verified. Numerical results are carried out to study the effect of some key parameters on the performance measures of the model. The vacation policy is analyzed in this paper is more general.

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